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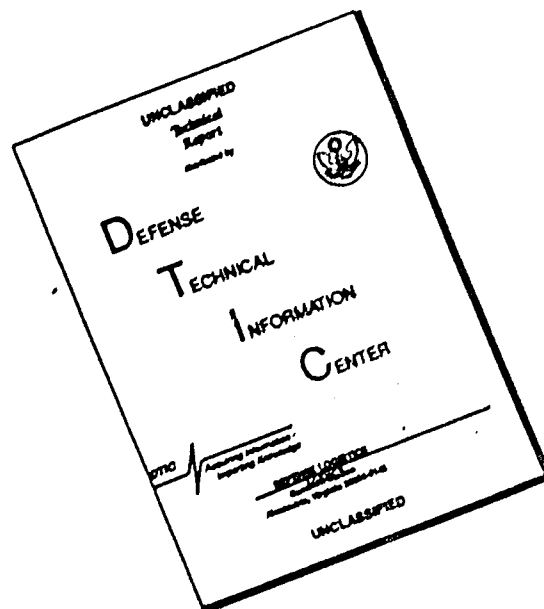
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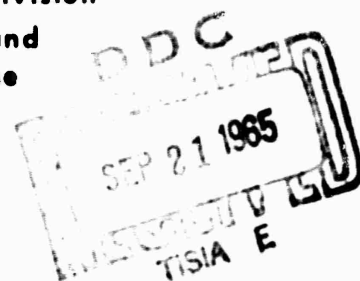
RADIATION-EFFECTS RELIABILITY AND DATA ANALYSIS TECHNIQUES

J. B. Wattier R. D. Ingram
L. J. York W. R. Owens

Nuclear Aerospace Research Facility
General Dynamics Corporation
Fort Worth, Texas
Contract Contract AF 29(601)-6308

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September 1965

AIR FORCE WEAPONS LABORATORY
Research and Technology Division
Air Force Systems Command
Kirtland Air Force Base
New Mexico



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FOREWORD

The work reported in this document was performed at the Nuclear Aerospace Research Facility, General Dynamics/Fort Worth, under Air Force Contract AF 29(601)-6308, Project 1831, Task 183108, Program Element 6.24.05.21.4. The document is submitted in accordance with the statement of work, Exhibit "A" to Purchase Request NR 146318, dated 21 August 1963. The work was performed during the period 23 March 1964 through 23 March 1965. The report was submitted 18 August 1965.

The Air Force Project Monitor is 1Lt R. L. Walker, AFWL (WLDN). The contractor's report number is FZK-242.

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ABSTRACT

The applicability of standard mathematical techniques for analysis of reliability of components exposed to a radiation environment is presented. The sensitivity of failure-distribution functions, data-presentation techniques, statistical parameters, and types of measurements to practical analysis methods is demonstrated. With the insight gained from the analysis methods, a research and development program was designed to establish qualified test procedures for long-term nuclear-development techniques. In addition, the lack of practical standard test techniques is substantiated and standard data-reporting methods are recommended.

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SECTION I

INTRODUCTION

Scientific interest in the relationship between radiation effects and reliability has increased with the advancement in space technologies and the advent of nuclear propulsion and nuclear auxiliary power. The long-term missions associated with future space objectives, especially those of a nuclear nature, have created a critical requirement for methods of predicting system response to long-life complex environments.

This study was initiated to determine the relationship between radiation effects and reliability of space-systems components and to investigate the applicability of various component-reliability predicting methods relative to the utilization of nuclear auxiliary power.

The main result of this study is the realization of the advantages of applying statistical techniques to preplanning, performing, and analyzing reliability studies. Therefore, the basic principles of statistical analysis are presented in detail, and methods of analysis are demonstrated by application to available radiation-effects and reliability data.

Present-day test techniques and military specifications are inadequate for implementing new long-term components and for performing practical development of long-term programs. A proposed program for developing long-term techniques is outlined, the lack of standard test techniques is pointed out, and data-presentation methods are suggested.

SECTION II

MATHEMATICAL MODELS ANALYSIS

Although a sensitivity analysis was performed simultaneously with the mathematical models analysis, the latter is presented first in order to familiarize the reader with the mathematical and statistical concepts essential to understanding the techniques and results presented in this report.

The objective of the mathematical models analysis was to investigate and develop analytical models that can be used to describe those relationships between radiation effects and reliability that were observed in the sensitivity analysis. To achieve this objective, the mathematical models analysis was performed in three phases:

Phase 1. Investigation of fundamental reliability concepts and relationships, possible radiation-effects reliability models, and reliability probability density functions.

Phase 2. Selection of the "best" family of failure distributions for use in describing the observed relationships between radiation effects and reliability; presentation of parameter estimation techniques for the selected failure distribution; and determination of the radiation-effects failure rate with the use of a statistical multiple-regression analysis.

Phase 3. Investigation of the consequences of using the commonly assumed exponential reliability distribution as a radiation-effects reliability model.

These three phases are described in detail in Sections 1, 2, and 3.

Section 1 describes (a) fundamental reliability concepts and the analytical relationships between probability density functions, failure-rate functions, and the resulting reliability equations;

(b) mathematical models which can be used to represent the reliability of electronic equipment operating in various nuclear environments; and (c) probability density functions which are considered here for their capability of representing the reliability of equipment subjected to a nuclear environment.

Section 2 presents engineering and statistical techniques for use in estimating the true, but unknown, failure distribution. The selection of the unknown failure distribution is based upon the analysis of empirical test data. The selection techniques presented are based upon the concept of discrimination between a priori failure distributions by means of engineering goodness-of-fit measures and statistical goodness-of-fit tests. The particular techniques considered are (a) graphical goodness-of-fit comparisons by the use of probability papers, (b) nonparametric and parametric statistical goodness-of-fit tests, and (c) comparison of the normalized residual sum of squares.

Also presented in Section 2 are the procedures for estimating the parameters of a Weibull distribution, with special emphasis given to the graphical technique, which is based upon the method of least squares. The placement of confidence intervals upon the Weibull cumulative density function is also considered. In addition, Section 2 presents a technique for determining the reliability function; this technique is based upon the use of a statistical multiple-regression analysis of the failure-rate function.

Section 3 considers the effects of an erroneous exponential assumption upon two factors when the true but unknown failure

distribution is a Weibull distribution with shape parameter $\beta > 1$.

These factors are:

- Reliability estimates based upon actual test data.
- The testing errors of any subsequently designed test program.

1. Reliability Concepts

a. Reliability Relationships

This subsection presents an exposition of the reliability concepts, equations, and relationships employed in this study. The purpose of this exposition is to clarify the reliability concepts and terminology that are used throughout this report.

The reliability of a device can be expressed in terms of any applicable random variable or variables; however, for purposes of exposition, it will be expressed as a function of the single random variable - time. Thus, reliability will be defined as the probability of a device operating within specified limits for the time and operating conditions specified.

The reliability of a device can be expressed in two equivalent ways: (1) in terms of a probability density function or (2) in terms of the equivalent failure rate. Although system reliabilities are almost always presented in terms of a failure rate, the commonly used techniques of reliability data analysis require a knowledge of the equivalent probability-density-function representation.

The following reliability relationships can be obtained by the application of elementary probability theory:*

*Throughout this report, unless otherwise specified, the range of t is $0 \leq t \leq \infty$

$$F(t) = \int_0^t f(x)dx \quad (1)$$

$$R(t) = 1 - F(t) \quad (2)$$

$$R(t) = \exp \left[- \int_0^t g(x)dx \right] \quad (3)$$

where

$F(t)$ = cumulative density function, or c.d.f.

$f(t)$ = probability density function, or p.d.f.

$R(t)$ = reliability function

$g(t)$ = failure rate function

If $f(t)$ is the p.d.f. of the time to failure, it gives the density of the probability at any point t ; for small Δt , $f(t)\Delta t$ is the probability of a device failing in the interval of time between t and $t + \Delta t$. Therefore, Equation 1 is equal to the probability that the device will fail in the time interval between 0 and t , and Equation 2 is equal to the probability that a device will not fail during the time interval between 0 and t . Further, the probability that a device will fail in the interval of time between t_1 and t_2 is

$$F(t_2) - F(t_1) = \int_{t_1}^{t_2} f(x)dx$$

The failure rate of a device, $g(t)$, gives the density of the conditional probability of failure at time t , given that the device has not failed prior to time t . Thus, for small Δt , $g(t)\Delta t$ represents the probability that a device which has not failed prior to time t will fail in the interval $(t, t + \Delta t)$.

The relationship between the failure p.d.f., $f(t)$, and the failure rate, $g(t)$, is easily shown to be

$$g(t) = \frac{f(t)}{1 - F(t)} \quad (4)$$

This equation expresses the failure rate in terms of the known p.d.f. and c.d.f. To express the reliability, $R(t)$, in terms of the failure rate, integrate the differential equation

$$g(t)dt = \frac{dF(t)}{1 - F(t)}$$

over the range 0 to t . By noting that $F(0) = 0$, the reliability, $1 - F(t)$, can be expressed in terms of the failure rate as given in Equation 3.

As illustrated above, the two methods of expressing reliability - in terms of the p.d.f. or the equivalent failure rate - are identical. If the underlying form of the p.d.f. or the equivalent failure rate and the values of their respective parameters were known, the consideration of a single approach would be sufficient. However, the radiation-effects reliability study is based on the analysis of empirical data and requires consideration of both approaches for two reasons:

1. The method of estimating (from test data) the parameters involved differs for the two approaches.
2. Each approach has advantages and disadvantages in various situations.

For example, in the p.d.f. approach the general procedure is to assume some well-known p.d.f. for $f(t)$, then estimate, by use of graphical procedures, method of moments, least squares, method

of maximum likelihood, or some other appropriate estimation technique, the parameters of $f(t)$ on the basis of empirical data. In the failure-rate approach, the assumption is made that there exists a transformation, $y(t)$, which transforms the random variable, time-to-failure t , into a random variable that is exponentially distributed, namely,

$$R(t) = e^{-y(t)} \quad (5)$$

This approach does not attempt to hypothesize the form of the underlying p.d.f. It merely assumes that the integral of the true failure rate, $y(t) = \int_0^t g(x)dx$, can be approximated by a polynomial of relatively low degree.

Obviously, in some cases either approach is possible and the resulting analytical functions are easily handled. However, there are many cases that can not be solved by one approach, but are workable in terms of the other approach.

b. Reliability Models

The mathematical models (composite, mixed, and failure rate) presented in this subsection express the relationship between radiation effects and reliability for various combinations of failure modes.

For purposes of model explanation, it will be advantageous to express reliability as a function of the single variable time, realizing that at any given point in time the failure rate - and, consequently, the reliability - is dependent upon the effect of two factors: random failures and radiation-induced failures. Thus, the total failure rate may be considered

as the result of two forces: (1) the random failure force, which is observed in equipment operating under normal environmental conditions; and (2) the radiation failure force, which is responsible for the failures that are caused solely by radiation effects. The obvious extension of these models to make them 2-dimensional for inclusion of the radiation effects is to consider the total radiation dose as the radiation-effects variable. The total dose received is, of course, dependent upon the radiation rate and the length of exposure. Thus, if rate is treated as a parameter, it is possible to express the effect of both failure forces - chance and radiation effects - as a function of time alone.

(1) The Composite Model

The r-factor composite model is defined as

$$F(t) = F_1(t)$$

where

$$\delta_1 \leq t \leq \delta_1 + 1 \quad \text{and} \quad i = 1, \dots, r$$

The δ_1 's are points of component partition or simply partition parameters. As illustrated in Figure 1, the composite model is capable of representing either a single failure distribution or a sequence of failure distributions. The partitioning factors coincide with changing failure forces or transition periods produced by a single failure force. Situations in which the composite model may be applicable are illustrated in Figure 1 and discussed below.

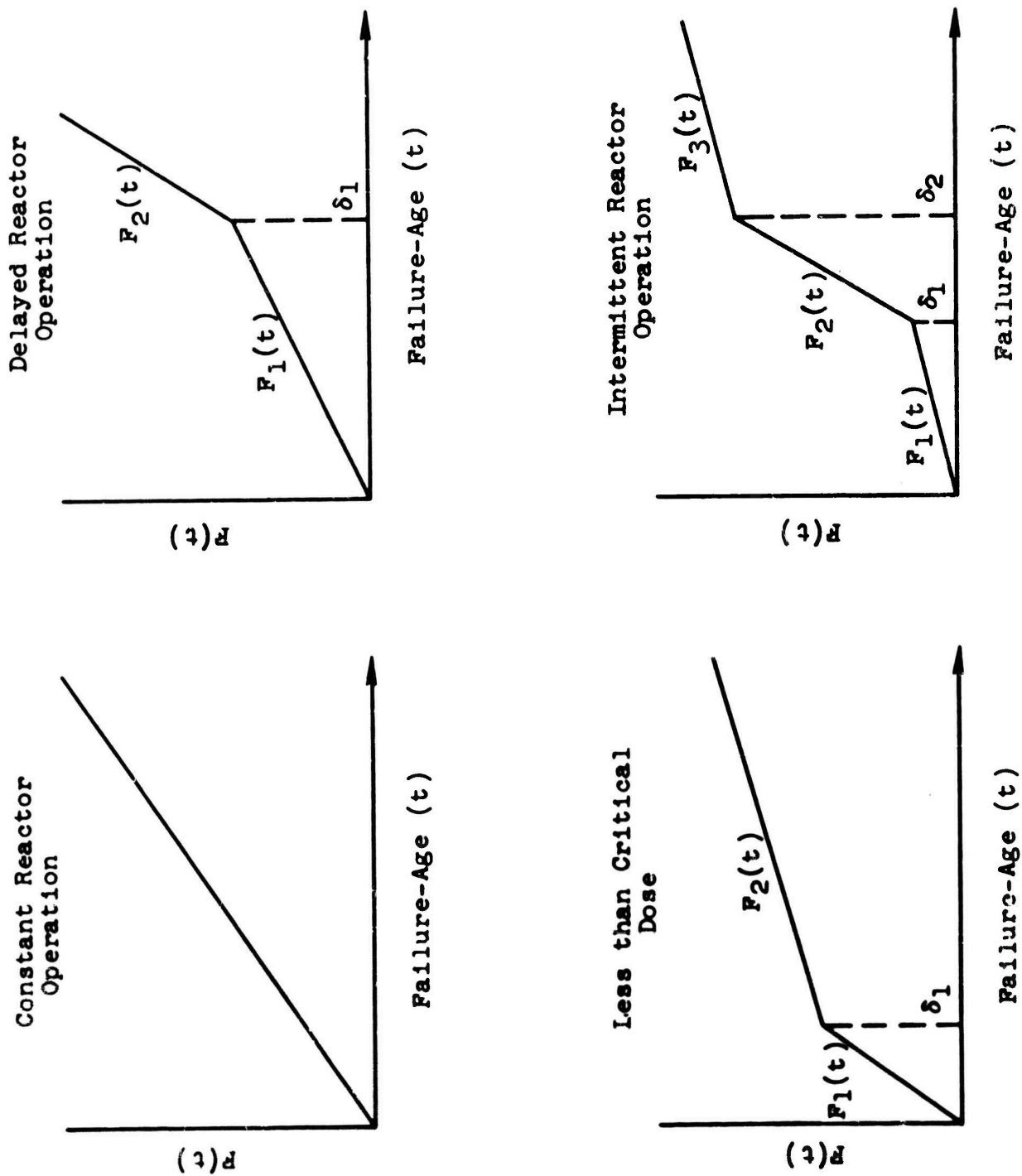


Figure 1 The Composite Reliability Model

1. Constant reactor operation. The equipment is subjected to a constant radiation environment and displays increasing failures with increased exposure time.
2. Delayed reactor operation. The equipment is operating in a normal environment until time δ_1 . At this time the reactor is turned on, introducing the radiation-effects failure force which increases the equipment failure rate.
3. Less than critical dose. The equipment is subjected to a less-than-critical radiation environment until time δ_1 . At this time the reactor is turned off and the equipment is operated in a normal environment.
4. Intermittent reactor operation. The equipment is subjected to varying radiation environments at intermittent intervals; consequently, the radiation failure force is present at varying levels.

The composite model can be used to describe changing failure distributions, as illustrated above. However, the assumption is made that at any point in time the single failure force or several failure forces do combine in such a manner that, within each partition, it is possible to describe the reliability relationship by a single failure distribution. Figure 2a illustrates the c.d.f. of a 2-component composite model.

(2) The Mixed Model

The 2-factor mixed model is defined as

$$F(t) = pF_1(t) + qF_2(t), \quad p + q = 1$$

where $F_1(t)$ is the c.d.f. of the i th subpopulation, and p and q are the mix percentages of subpopulations 1 and 2, respectively.

The mixed model is proposed as a radiation-effects failure distribution in which $F_1(t)$ and $F_2(t)$ are the model components that account for or represent the cumulative failure distributions for chance failures and radiation-effects failures.

Figure 3 is an illustration of the subpopulation components and

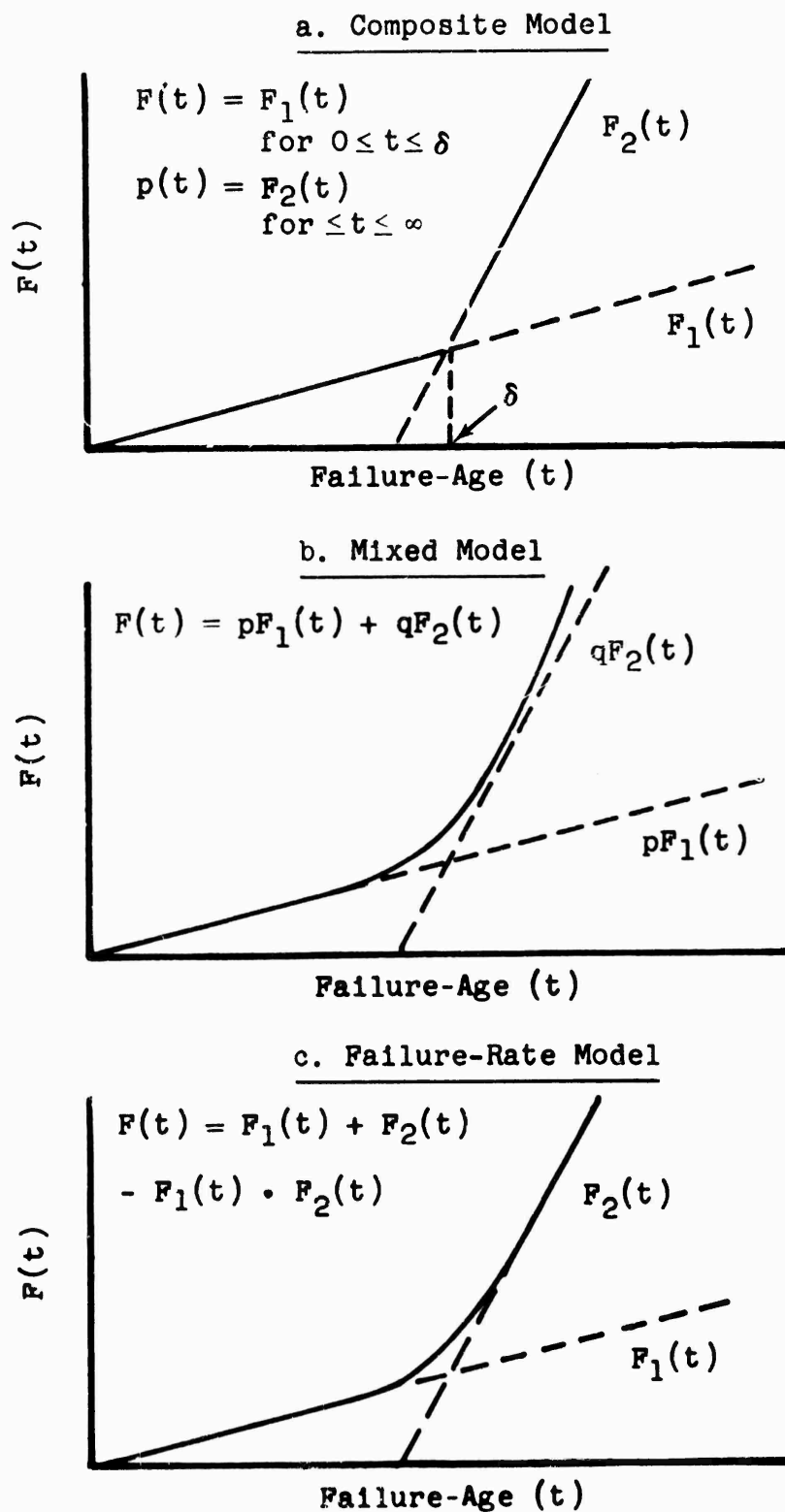


Figure 2 Schematic Plots of the Composite, Mixed, and Failure-Rate Models

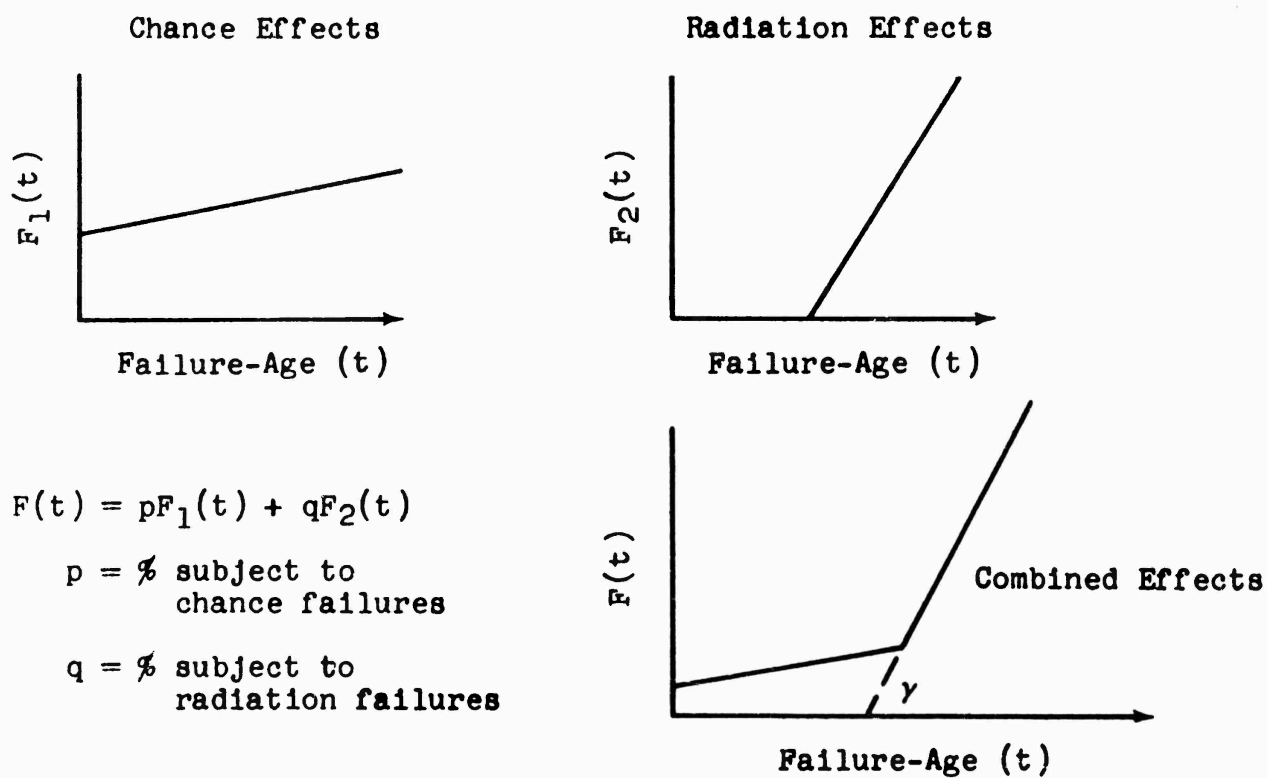


Figure 3 The Mixed Reliability Model

the results of their mixture. The applicability of the mixed model as a radiation-effects reliability model can best be evaluated by considering the development or theoretical basis of the model. The mixed model can be considered as being derived from the solution of the problem described below.

A population of components is to be developed by accumulation of components from a production line. The probability of obtaining a defective component upon random selection from the production process is equal to p . The probability of obtaining a nondefective component upon random selection from the production process is equal to q . The so-called defective items are subject only to chance failures, and the so-called nondefective items are subject only to radiation-induced failures. It is further known that the defective and nondefective items have cumulative failure distributions $F_1(t)$ and $F_2(t)$, respectively. If a population of components is gathered from the above production process and operated in a radiation environment, the following question arises: What is the failure distribution of the combined population?

From elementary probability theory, it is known that the resultant probability of a component failure on or before time t is equal to the product of the probability of drawing a defective component times the probability that the defective component fails on or before time t , plus the probability of drawing a nondefective component times the probability that it fails on or before time t . Thus, the failure distribution of the combined population is

$$F(t) = pF_1(t) + qF_2(t)$$

In summary, $F_i(t)$ is the c.d.f. of the i th subpopulation. The quantities p and q are the proportions of the subpopulations mix, or simply the mix parameters. This model combines two subpopulations with known failure distributions in the proportions p and q to establish the failure distribution of the mixed population.

Situations in which the mixed model may be applicable are presented below:

1. Mutually exclusive failure effects. If a lot of components consist of p percent "defective" components, say off-the-shelf items, and q percent "nondefective" components, say radiation-hardened items, the mixed model is appropriate for describing the failure distribution of this lot when operated in a radiation environment.
2. Combined batches or manufacturers. If a lot of components is a combination from two different batches or manufacturers, with independently determined failure distributions, the mixed model is appropriate for describing the resulting failure distribution.

Figure 2b illustrates the c.d.f. of a 2-component mixed model.

(3) The Failure-Rate Model

The failure-rate model is defined as

$$F(t) = 1 - \exp \left[- \int_0^t g(x) dx \right]$$

where $g(x)$ is the failure rate.

The failure rate for a 2-factor, chance-plus-radiation-effects, failure-rate model can be expressed in the following form:

$$g(t) = g_1(t; C) + g_2(t; R) + I(t_C, t_R)$$

where

$g_1(t; C)$ is the contribution of chance failures to the total failure rate and is dependent upon the

type of equipment and the length of time the equipment has been operating.

$g_2(t;R)$ is the contribution of radiation-effects failures to the total failure rate and is dependent upon the radiation environment and the length of time the equipment has been operating.

$I(t_C, t_R)$ is the contribution of the interaction effects that result from combining the chance and radiation-effects failure forces.

C and R denote the parameter vectors associated with the chance and radiation-effects failure rates, respectively.

The salient features of this model are:

1. It has the capability of representing reliability models that can be expressed in terms of a single failure distribution.
2. It is extremely flexible in the synthesis of a reliability model when the equipment under study is subject to more than a single failure force, but the only data available are those that describe the failure distributions of the equipment operating, subject to only one failure force at a time.
3. It automatically eliminates the problem of normalization that is encountered when combining probability density functions directly.

Figure 2c illustrates the c.d.f. of a 2-factor failure-rate model for which the interaction failure-rate component is zero.

c. Reliability Distributions

Several distributions (Weibull, exponential, Rayleigh, extreme-value, truncated-normal, and log-normal) were studied to determine their applicability and versatility for representing component reliability in a radiation environment. Since these distributions are being used to represent the operating life of various components, the variable in question is a non-negative number. Consequently, in the work that follows the

range of t is $t \geq 0$. To relocate a failure distribution to some origin greater than zero - to account for a so-called guarantee period (that is, a failure cannot occur before a certain time has elapsed) - the location parameter gamma, $\gamma \geq 0$, is introduced. Then, of course, the range of t becomes $t \geq \gamma$. The p.d.f.'s, their parameters, and relationships to reliability, as discussed in Section II-1a, are presented below.

(1) The Weibull Distribution

The Weibull cumulative density function is defined as

$$F(t) = 1 - \exp \left[- \frac{(t - \gamma)^\beta}{\alpha} \right]$$

where

$$t \geq \gamma, \quad \gamma \geq 0, \quad \alpha, \beta$$

and

α = the scale parameter

β = the shape parameter

γ = the location parameter

The Weibull probability density function is

$$f(t) = \frac{\beta(t - \gamma)^{\beta-1}}{\alpha} \exp \left[- \frac{(t - \gamma)^\beta}{\alpha} \right]$$

If the failure distribution of a component can be described by a Weibull distribution, the reliability function of Equation 2 is

$$R(t) = 1, \quad t < \gamma$$

$$R(t) = \exp \left[- \frac{(t - \gamma)^\beta}{\alpha} \right], \quad t \geq \gamma$$

Upon substitution of the Weibull c.d.f. and p.d.f. into

Equation 4, it is seen that the failure rate, $g(t)$, for a Weibull distribution is a decreasing, constant, or increasing function, depending upon the value of β :

$$g(t) = \frac{\beta(t - \gamma)^{\beta-1}}{\alpha}$$

Expressing reliability in terms of the failure-rate approach gives identical results:

$$\begin{aligned} R(t) &= 1, & t < \gamma \\ R(t) &= \exp \left[- \int_0^t \frac{\beta(x - \gamma)^{\beta-1}}{\alpha} dx \right], & t \geq \gamma \\ R(t) &= \exp \left[- \frac{(t - \gamma)^\beta}{\alpha} \right], & t \geq \gamma \end{aligned}$$

One of the factors contributing to the popularity of the Weibull distribution as a failure distribution for use in reliability work is its versatility. For example, the Weibull distribution encompasses the exponential and Rayleigh distributions. These distributions are special cases of the Weibull distribution which can be achieved by setting the shape parameter β equal to 1 and 2, respectively. The family of exponential distributions is by far the best known and most thoroughly explored distribution in reliability work.

The exponential distribution has a number of desirable mathematical properties, but its applicability to radiation-effects reliability work is limited because of its constant failure rate. For a constant failure rate to be applicable, previous equipment operation and exposure to a radiation environment must not affect the equipment's future life.

The Exponential Distribution ($\beta = 1$). The 2-parameter exponential distribution is a Weibull distribution with shape parameter $\beta = 1$. The c.d.f., p.d.f., and failure rate of this distribution are presented below:

$$F(t) = 1 - \exp \left[- \frac{(t - \gamma)}{a} \right]$$

$$f(t) = \frac{1}{a} \exp \left[- \frac{(t - \gamma)}{a} \right]$$

$$g(t) = \frac{1}{a}$$

The Rayleigh Distribution ($\beta = 2$). The 2-parameter Rayleigh distribution is a Weibull distribution with shape parameter $\beta = 2$. The c.d.f., p.d.f., and failure rate of this distribution are presented below:

$$F(t) = 1 - \exp \left[- \frac{(t - \gamma)^2}{a} \right]$$

$$f(t) = \frac{2(t - \gamma)}{a} \exp \left[- \frac{(t - \gamma)^2}{a} \right]$$

$$g(t) = \frac{2(t - \gamma)}{a}$$

(2) The Extreme-Value Distribution

A modification of the extreme-value c.d.f. is defined as

$$F(t) = 1 - \exp \left\{ - a \left[\exp(t - \gamma) - 1 \right] \right\}$$

where

$$a > 0, \quad t \geq \gamma, \quad \gamma \geq 0$$

and

a is the scale parameter

γ is the location parameter.

The modified extreme-value p.d.f. is

$$f(t) = \alpha \exp(t - \gamma) \exp \left\{ -\alpha \left[\exp(t - \gamma) - 1 \right] \right\}$$

Substitution of these two equations into Equation 4 gives the failure rate for the modified extreme-value distribution:

$$g(t) = \alpha \exp(t - \gamma).$$

(3) The Truncated Normal Distribution

The truncated normal p.d.f. is

$$f(t) = \frac{1}{c \sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{t - \mu}{\sigma} \right)^2 \right]$$

$$\sigma > 0, \quad 0 \leq \mu < \infty, \quad t \geq 0$$

where c is a normalizing constant defined as

$$c = \frac{1}{\sigma \sqrt{2\pi}} \int_0^{\infty} \exp \left[-\frac{1}{2} \left(\frac{t - \mu}{\sigma} \right)^2 \right] dt$$

and

σ is the scale parameter

μ is the location parameter.

(4) The Log-Normal Distribution

The log-normal p.d.f. is

$$f(t) = \frac{1}{t \sigma \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\log t - \mu}{\sigma} \right)^2 \right],$$

$$\sigma > 0, \quad 0 \leq \mu < \infty, \quad t \geq 0$$

The failure rates for the truncated-normal and log-normal distributions are rather complicated expressions; consequently,

the failure rate approach is not recommended when dealing with these two distributions. Further generalizations of these two distributions to account for the so-called guarantee period can be achieved by replacing t by $(t - \gamma)$, where γ is the guarantee time.

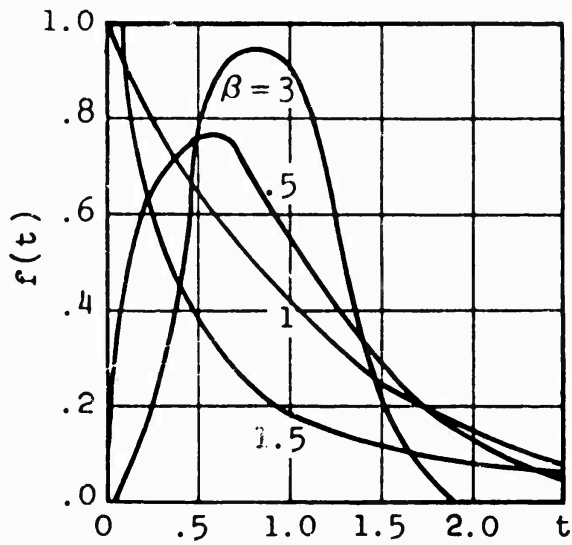
(5) Graphical Presentation of Reliability Distributions for Various Parameters and Forms

A graphical illustration of the p.d.f.'s, c.d.f.'s, and failure rate forms of reliability models discussed above are presented in Figures 4, 5, and 6. This presentation is used to graphically illustrate the reliability models and how they change for various input parameters. The reliability models differ mathematically, but as can be seen from Figures 4, 5, and 6, the graphical presentations are quite similar, especially for the c.d.f.'s in Figure 5.

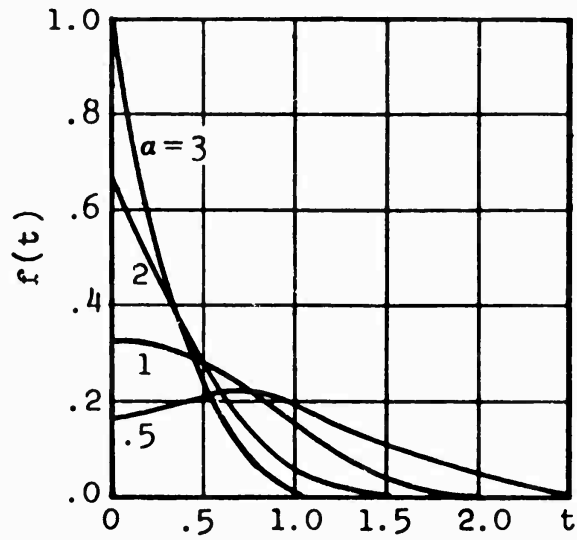
2. Methodology and Results of the Mathematical Models Failure Data Analysis

One of the basic problems encountered in any failure data analysis is that of verifying or rejecting the various a priori* failure distributions. Selection of the appropriate failure distribution and the associated techniques of failure data analysis are presented in this subsection. Because of the display and use of radiation-effects reliability failure data throughout this subsection, the random variable time t is replaced with that of radiation dose d .

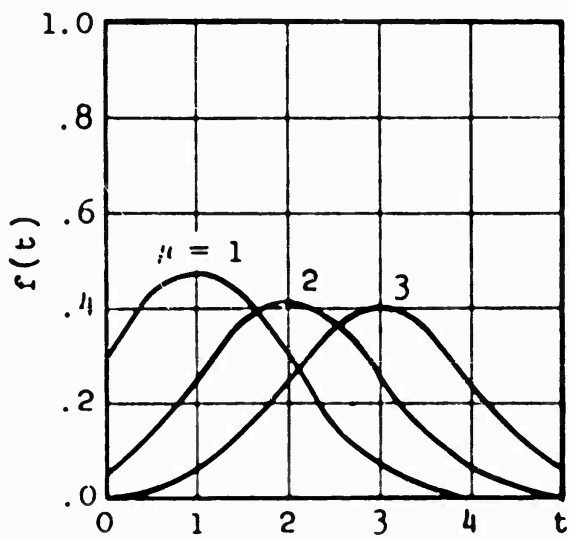
* a priori Determined in advance - before the fact - as opposed to a posteriori, determined in retrospect - after the fact.



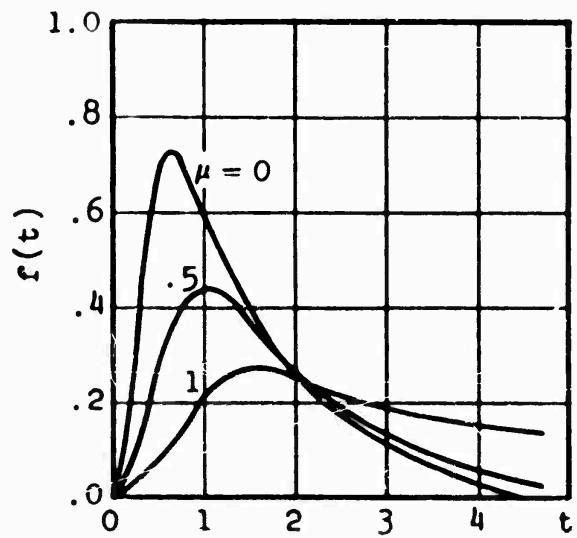
Weibull Distribution
($\gamma = 0$, $\alpha = 1$)



Extreme-Value Distribution
($\gamma = 1$)



Truncated-Normal Distribution
($\sigma = 1$)



Log-Normal Distribution
($\sigma = .5$)

Figure 4 Reliability: Probability Density Functions

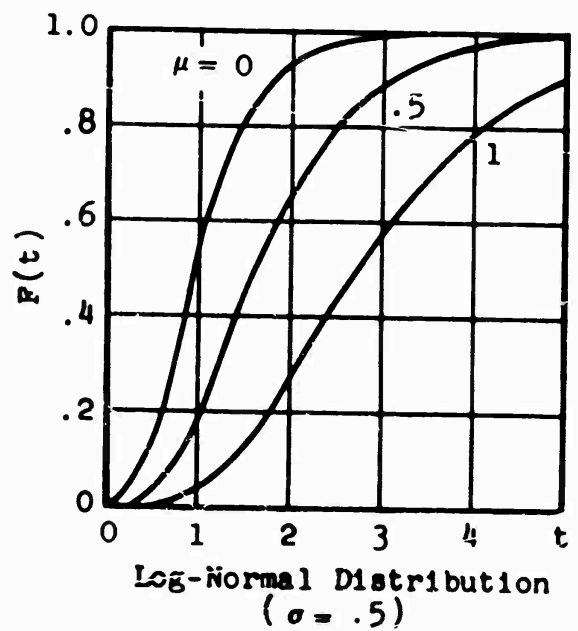
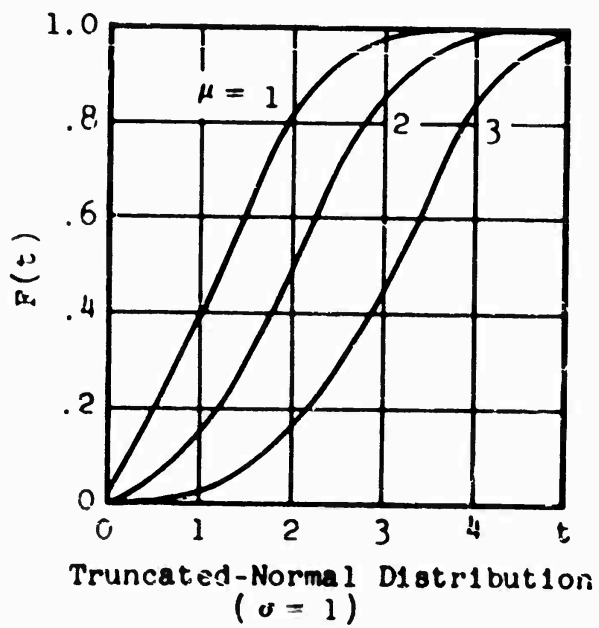
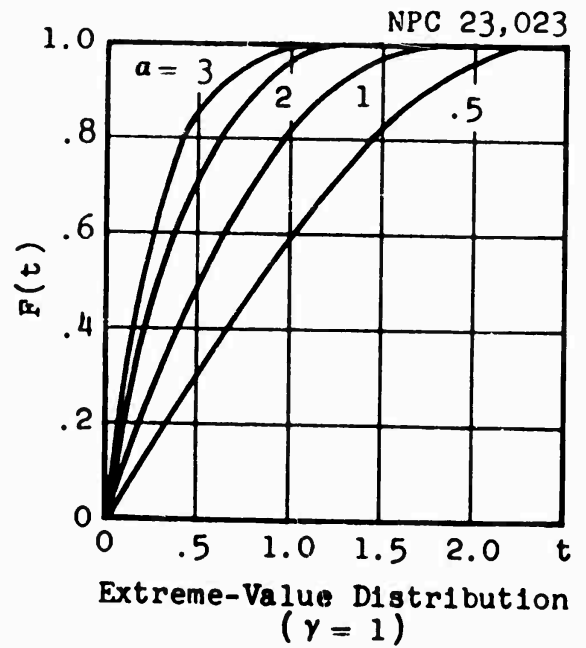
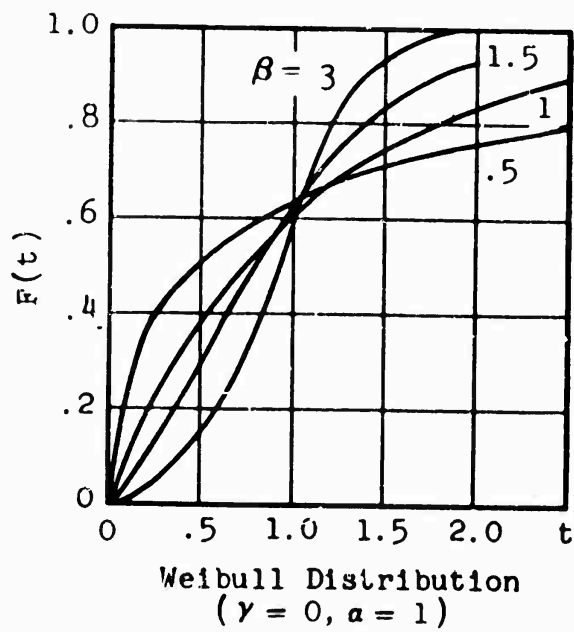
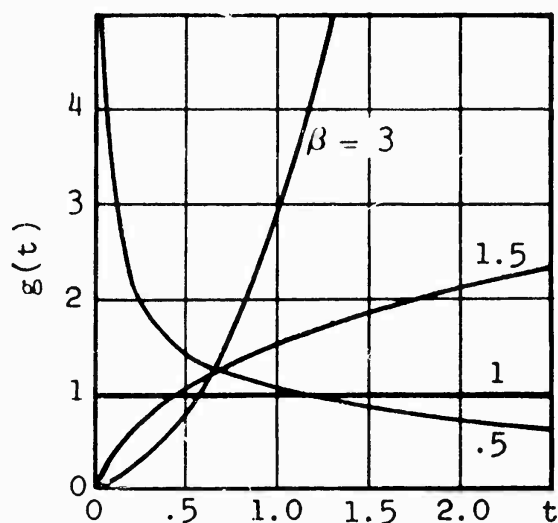
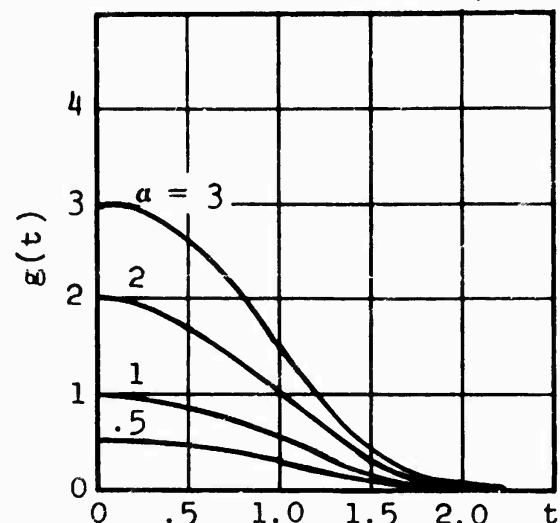


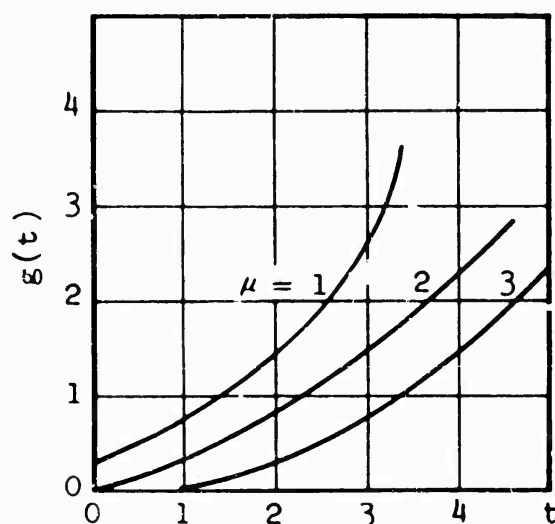
Figure 5 Reliability: Cumulative Density Functions



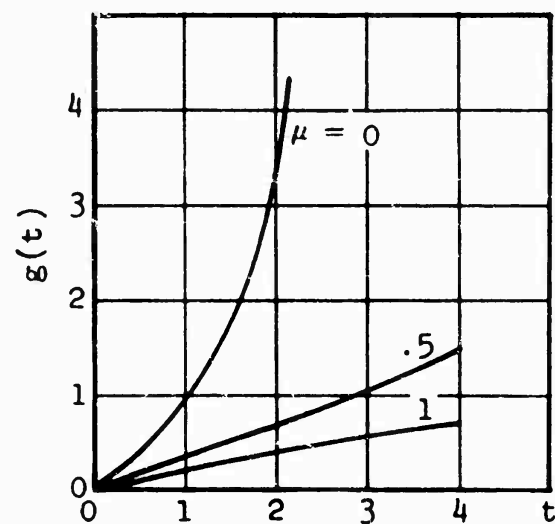
Weibull Distribution
($\gamma = 0$, $\alpha = 1$)



Extreme-Value Distribution
($\gamma = 1$)



Truncated Normal Distribution
($\sigma = 1$)



Log Normal Distribution
($\sigma = .5$)

Figure 6 Reliability: Failure-Rate Functions

a. Selection of a Reliability Model

The radiation-effects reliability models presented in Section II-1b are expressed in terms of arbitrary cumulative density functions and failure rates. Section II-1c presents various probability density functions that are frequently used to represent life-test data. Since the reliability models of Section II-1b consist of individual and combinations of the individual probability density functions of Section II-2c, the suggested approach to model selection is:

1. Determine the p.d.f. or failure rate of the equipment within the various regions of partition, subpopulations, or range of parameter interest.
2. Combine these results as indicated in the reliability models, using that model which fits the particular situation at hand.

b. Selection of a Reliability Distribution

An important step in any empirical reliability analysis is the mathematical formulation of the underlying failure distribution. Occasionally the form of the underlying failure distribution may be derived from knowledge of the physics of the materials involved; but, as a rule, a distribution type must be selected on the basis of empirical data. However, the distribution of the population cannot be uniquely determined by a set of empirical data. In fact, it is quite often possible to describe the same set of empirical data with several different mathematical distributions. Although the models may differ mathematically when they are utilized for a given set of observational data, they lead to distribution functions

whose graphical representations are almost identical; this is especially true when dealing with small sample sizes. The procedures for accepting or rejecting any hypothesized distribution as being the true underlying failure distribution are based upon some type of goodness-of-fit test or measure. When possible, the selection of the failure distribution should be based upon the results of a statistically valid goodness-of-fit test. However, this is frequently impossible and consequently the selection must be based upon the comparison of various goodness-of-fit measures.

(1) The Use of Probability Paper

By using a special type of graph paper, commonly called probability paper, it is possible to determine graphically the fit of a set of sample data to the various forms of failure distributions presented in Section II-1c. Probability paper is so constructed that a plot of the random variable d versus the theoretical cumulative density function $F(d)$ will produce a straight line. Consequently, if a set of data is actually a random sample from a specified failure distribution, a plot of these data on the probability paper of the specified distribution should approximate a straight line. Thus, by plotting a single set of sample data on the probability paper of each failure distribution being considered, a graphical goodness-of-fit comparison is performed, and the selection of a particular paper (on the basis of the best approximating straight line) is identical with the choice of a failure

distribution. A graphical goodness-of-fit comparison of the probability density functions of Section II-1c follows. For this comparison the radiation-effects reliability data presented in Table I are plotted in Figures 7 through 11 on probability paper representing each failure distribution.

In those cases where a reasonable fit is obtained on the probability paper, one can obtain estimates of the parameters of the underlying distribution from the properties of the resulting straight line. (For the general theory of parameter estimation by the use of probability paper see Reference 1.)

(2) Goodness-of-Fit Analyses

Briefly stated, statistical goodness-of-fit tests involve specifying a priori some cumulative density function and comparing it to an empirical cumulative density function. The degree of similarity between the observed and the theoretical distributions is then used as a basis for accepting or rejecting the a priori failure distribution.

(3) Nonparametric Goodness-of-Fit Tests

The most commonly used goodness-of-fit test is the Chi-square test. Use of the Chi-square test is often not possible, however, because of its large-sample-size requirements. In the present study, for example, the small sample sizes encountered did not meet the requirements of the Chi-square test for the minimum expected frequencies per subdivision.

The Kolmogorov-Smirnov was the test selected for use as a goodness-of-fit test in this study (Ref. 2). This test treats

Table I
2N1613 TRANSISTORS TESTED IN A
RADIATION ENVIRONMENT^a

Rank i	Percent Failure $F(d)$ $i/(n+1)$	Failure Dose $d \times 10^{-4}$ -10% Change	Failure Dose $d \times 10^{-4}$ -20% Change	Failure Dose $d \times 10^{-4}$ Transformed ^b -20% Change
1	0.091	0.66	5.30	0.30
2	0.182	1.39	5.39	0.39
3	0.273	1.39	5.76	0.76
4	0.364	1.39	6.12	1.12
5	0.455	2.12	6.40	1.40
6	0.546	2.84	6.50	1.50
7	0.637	2.84	7.20	2.20
8	0.728	3.21	7.31	2.31
9	0.819	3.21	7.94	2.94
10	0.910	3.57	9.40	4.40

^aData are from NARF-LMSC tests where h_{FE} , DC transistor gain, was obtained for collector to emitter voltage, $V_{CE} = 10$ v, and collector current, $I_C = -1$ ma. The two failure criteria are for -10% and -20% change in h_{FE} . The number of transistors tested was $N = 10$.

^bThe data in this column are a transformation of the data in column 4. The transformation is $(d_i - \hat{\gamma})$, where d_i is the gamma dose at the i th failure recorded in column 4 and $\hat{\gamma} = 5.0 \times 10^4$ rad(C) gamma dose. (See page 52 for the method of estimating γ .) The $(-)$ notation is used to denote an estimate of the true parameter in the mathematical model.

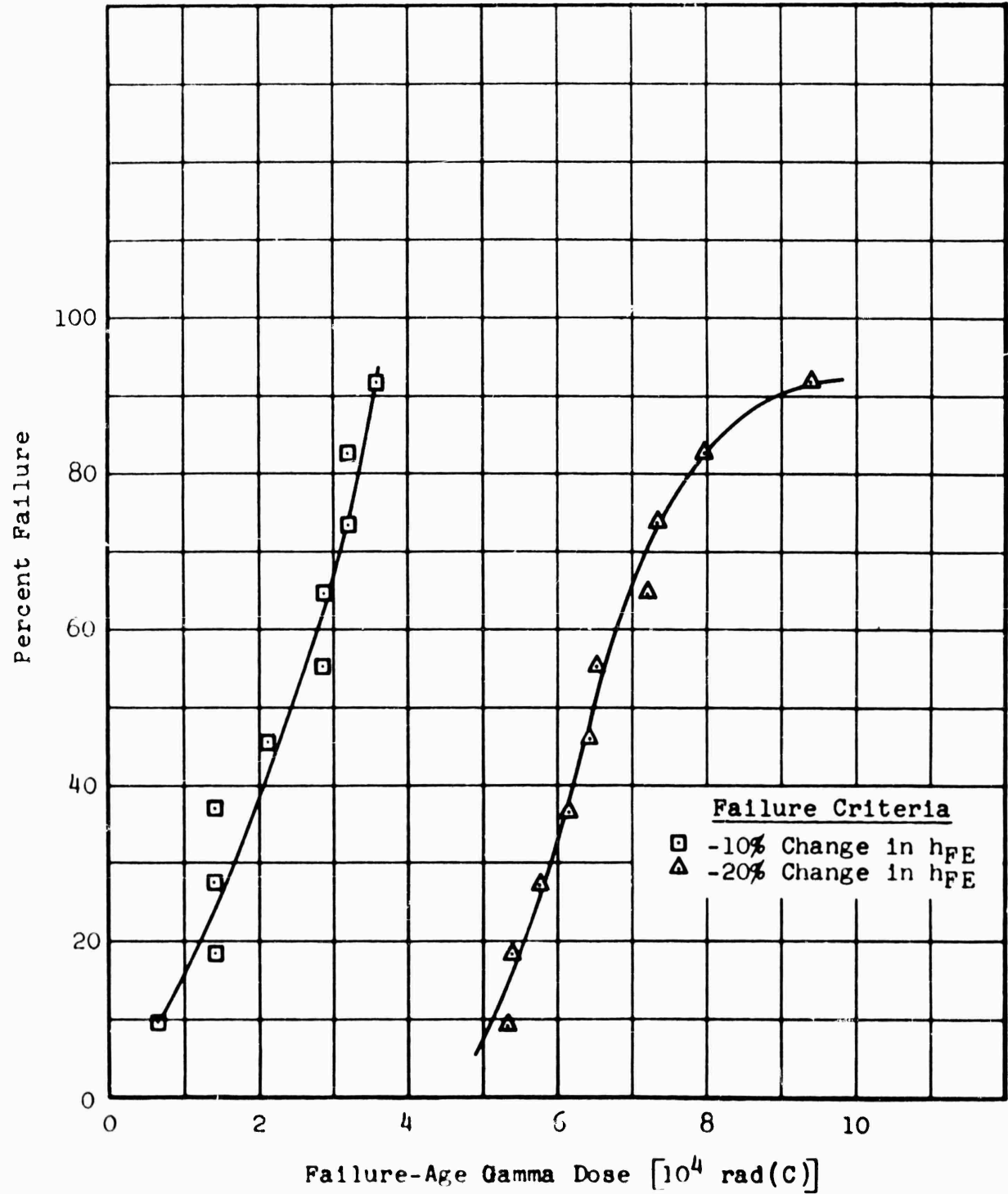


Figure 7 Linear Radiation-Reliability Plot of h_{FE} for 2N1613 Transistors

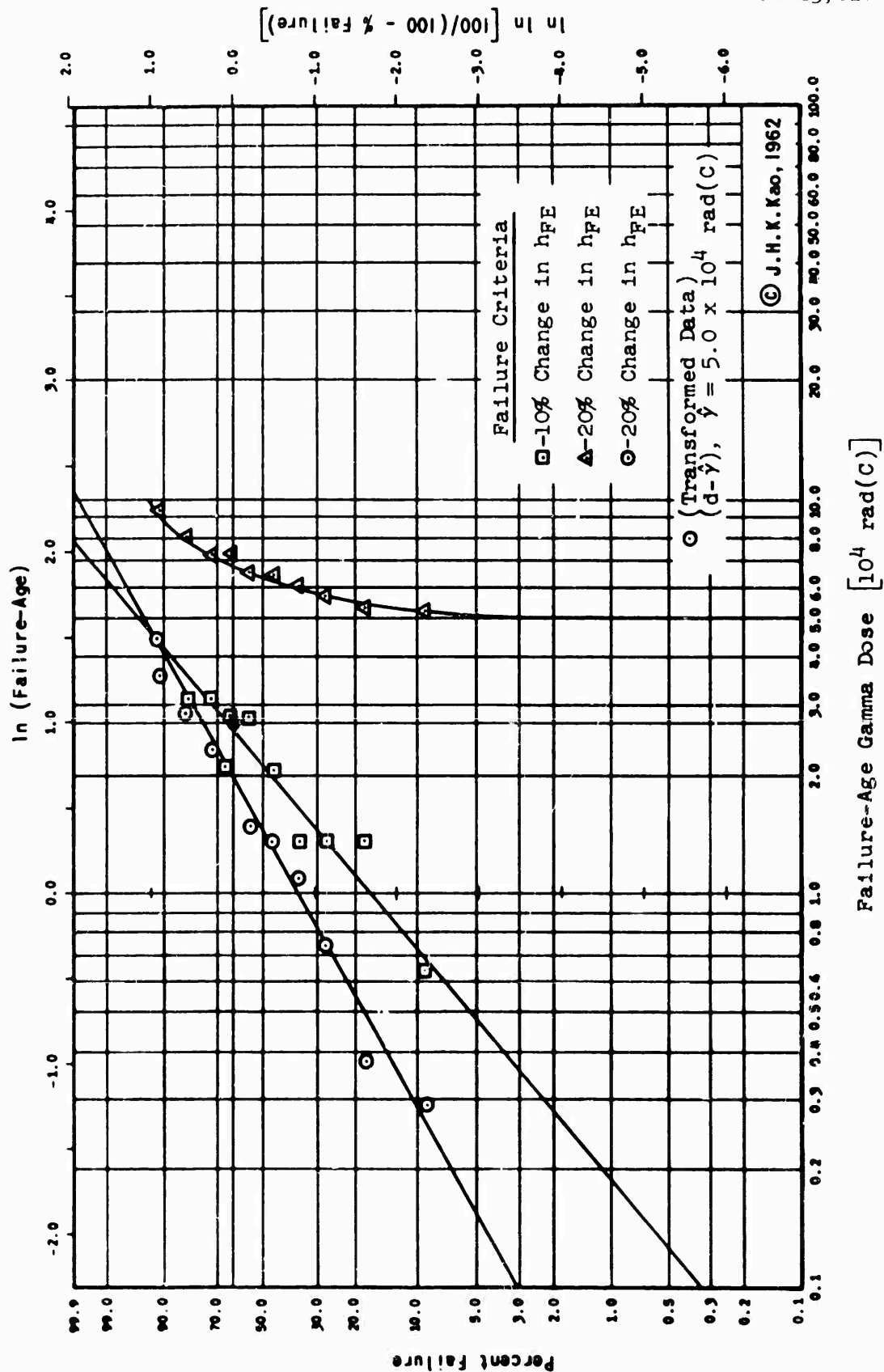


Figure 8 Weibull Radiation-Reliability Plot of h_{FE} for 2N1613 Transistors

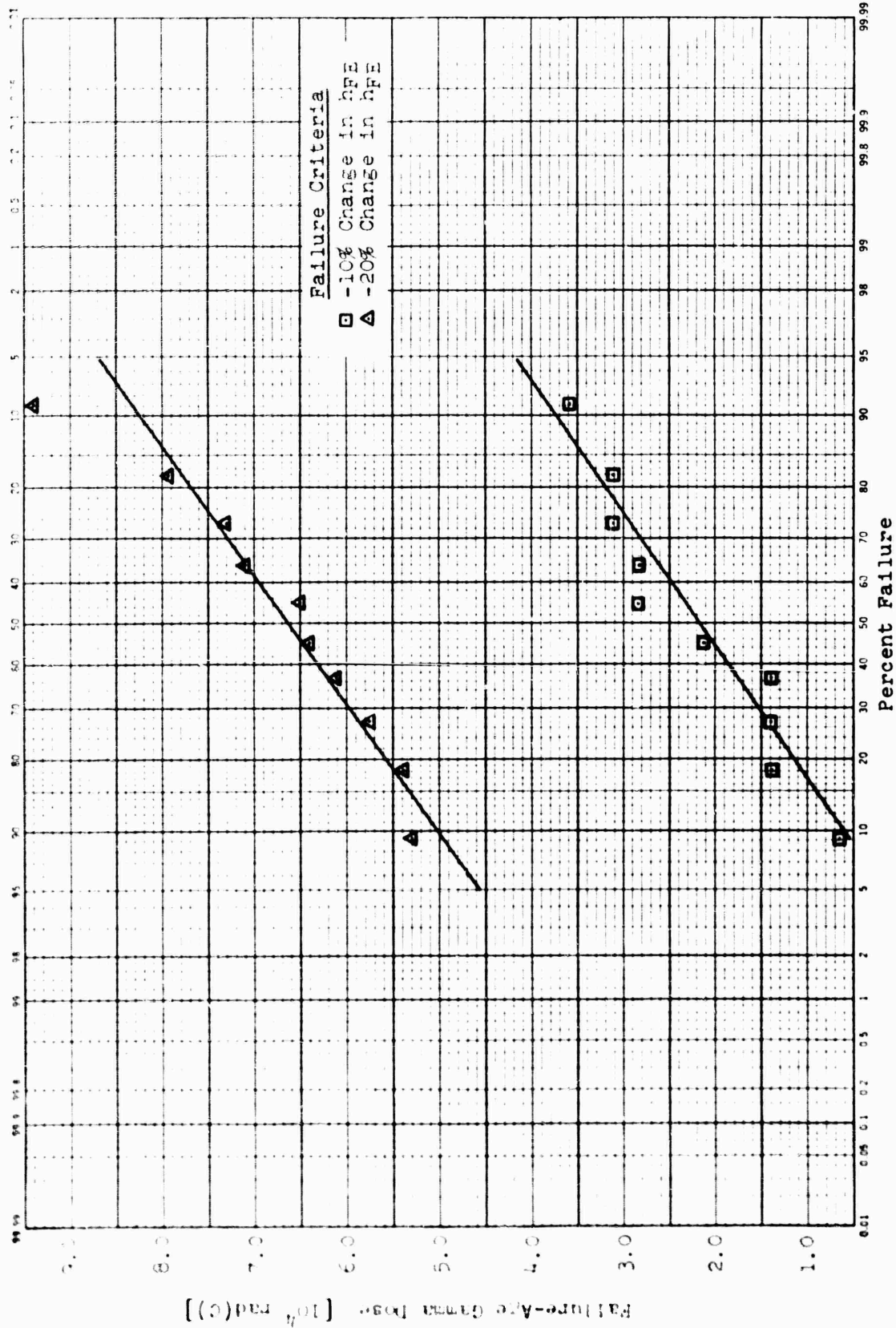


Figure 9 Normal Radiation-Reliability Plot of h_{FE} for 2N1613 Transistors

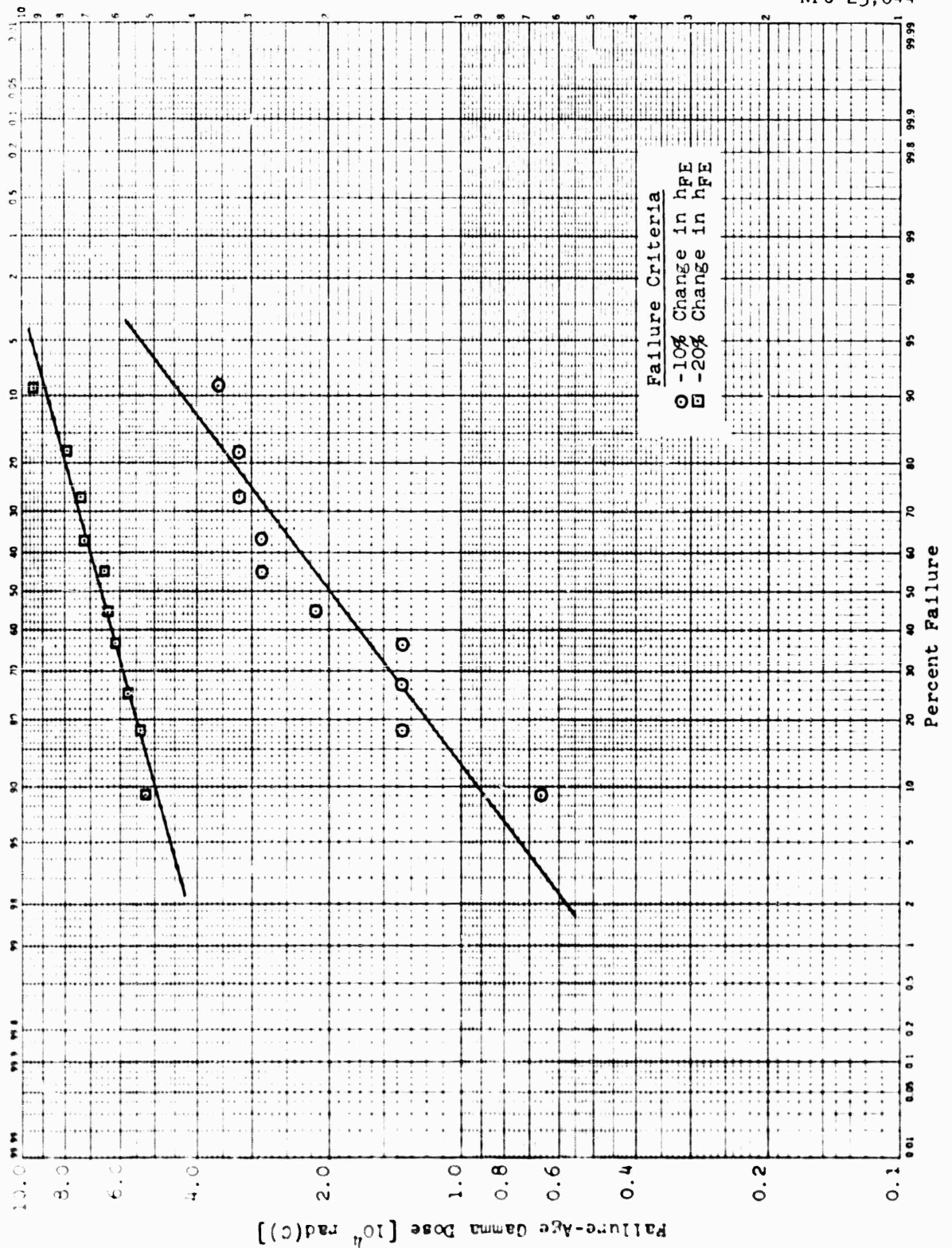


Figure 10 Log-Normal Radiation-Reliability Plot of h_{FE} for 2N1613 Transistors

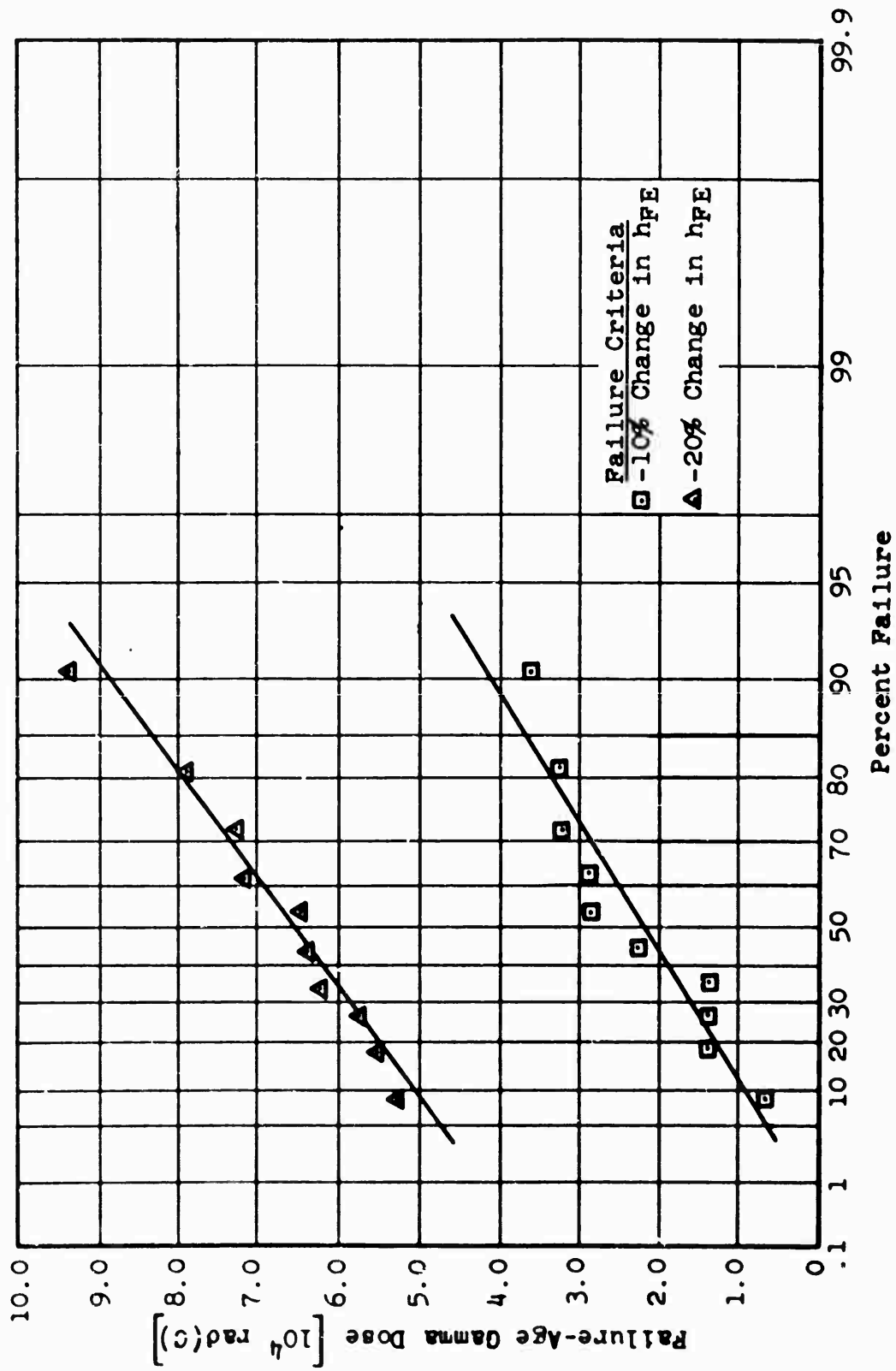


Figure 11 Extreme-Value Radiation-Reliability Plot of h_{FE} for 2N1613 Transistors

each observation individually and consequently may be used for small sample sizes. It is based on the sampling distribution of the maximum deviation, D , between the a priori and the observed cumulative density functions, i.e.,

$$D = \text{maximum} |F(d) - O(d)| ,$$

where $F(d)$ and $O(d)$ represent the cumulative density functions of the theoretical and observed failure distributions, respectively. Table II summarizes the results obtained from applying the Kolmogorov-Smirnov goodness-of-fit test to the radiation-effects reliability data of Table I. As indicated in Table II this test was applied to each of the failure distributions presented in Section II-1c.

As seen in Table II none of the a priori failure distributions can be rejected on the basis of the Kolmogorov-Smirnov test. The only a priori distribution for which the sample statistic D is even close to the critical region is the exponential distribution. In view of this failure to reject any of the a priori failure distributions, it should be noted that the Kolmogorov-Smirnov test is the most powerful (i.e., most discriminating) of the nonparametric tests. Consequently, its failure to reject any of the a priori failure distributions can be attributed to the small sample sizes involved.

(4) Parametric Goodness-of-Fit Test

The most powerful class of goodness-of-fit tests consists of parametric tests that are designed for specific distributions.

Table II
SUMMARY OF THE NONPARAMETRIC
GOODNESS-OF-FIT TEST

A Priori Failure Distribution	Observed Sample Statistic D ^a		80% Level Of Significance for Test Statistic D
	-10% Change in h_{FE}	-20% Change in h_{FE}	
Weibull ^b $\beta > 1$	0.144	0.042 ^c	0.322
Exponential	0.270	0.084	0.322
Truncated Normal	0.104	0.076	0.322
Log Normal	0.150	0.070	0.322
Extreme Value	0.154	0.071	0.322

^aIn order to reject the hypothesis (at the 80% level of significance) that the underlying failure distribution is of a specified type, the observed sample statistic D must be equal to or greater than $D_{80\%} = 0.322$ (see Ref. 2).

^bThe shape parameter of the Weibull distribution, β , was determined a posteriori.

^cBased on analysis of the transformed data $\left[\dot{\gamma} = 5.0 \times 10^4 \text{ rad(c)} \right]$.

The parametric tests found to be most suitable for the small sample sizes encountered were (a) the Geary and Pearson tests for normality and log normality and (b) a test presented by Epstein for exponentiality.

As a goodness-of-fit test for normality, the Geary and Pearson test of skewness and kurtosis is suggested (Ref. 3). There is a series of such tests for skewness and kurtosis; however, the statistic

$$S = \frac{\frac{1}{n} \sum_{i=1}^n |d_i - \bar{d}|}{\left[\frac{1}{n} \sum_{i=1}^n (d_i - \bar{d})^2 \right]^{1/2}}$$

was chosen as being the most appropriate for use in this study because of sample sizes and power considerations. The Geary and Pearson test is based on the fact that if the observations do come from a normally distributed population, then the sampling distribution of the statistic S is known. This statistic can also be used as a goodness-of-fit test for sample data believed to have come from a log-normal population. This is accomplished by taking the logarithms of the sample data before computing the statistic S . The upper and lower 1%, 5%, and 10% points for the sampling distribution of S can be obtained from Table 34 of Reference 4.

Table III summarizes the results obtained from applying

Table III
SUMMARY OF THE PARAMETRIC
GOODNESS-OF-FIT TEST

A Priori Failure Distribution	Observed Sample Statistic S or Z		90% Level of Significance for Test Statistic S or Z
	-10% Change in h_{FE}	-20% Change in h_{FE}	
Exponential	$Z = 2.48^a$	$Z = 4.75^a$	$Z_{90\%} = 1.65$
Truncated Normal	$S = 0.92^a$	$S = 0.81$	$S_{90\%} = 0.889$
Log Normal	$S = 0.87^a$	$S = 0.82$	$S_{80\%} = 0.867$

^aIndicates those cases for which the sample statistic is greater than the critical value, for the test under consideration, and consequently those cases for which the a priori failure distribution can be rejected at the indicated level of significance.

the Geary and Pearson test for normality and log normality to the radiation-effects reliability data of Table II. In Reference 5 Epstein presents several procedures for testing the validity of the assumption that a set of sample observations comes from an exponential distribution. Epstein's Test No. 3 has been selected for use in this study on the basis of sample size and power considerations. This test is based on the fact that if the observations do come from an exponential distribution, then the statistic Z, defined below, is an approximate standard normal deviate. With d_1 as the failure dose and r as the number of failures that have occurred on or before termination of the test, statistic Z is defined as

$$Z = \frac{\sum_{i=1}^{r-1} d_i - \left[\frac{(r-1)}{2} \cdot d_r \right]}{d_r \cdot \left[\frac{r-1}{12} \right]^{1/2}}$$

Table IV summarizes the results obtained from applying Epstein's Test No. 3 for the exponential distribution to the radiation-effects reliability data of Table I.

(5) Comparison of the Normalized Residual Sum of Squares

A statistic that can be used to measure the goodness-of-fit between an a priori cumulative density function and a sample cumulative density function is the normalized residual sum of

Table IV
SUMMARY OF THE NORMALIZED RESIDUAL
SUM OF SQUARES COMPARISONS

A Priori Failure Distribution	Observed Sample Statistics NRSS	
	-10% Change in h_{FE}	-20% Change in h_{FE}
Weibull ^a $\beta > 1$	0.477	0.039 ^b
Exponential	11.488	0.092
Truncated Normal	0.570	0.143
Log Normal	0.723	0.102
Extreme Value	0.503	0.050

^aThe shape parameter of the Weibull distribution, β , was determined a posteriori.

^bBased on analysis of the transformed data $\left[\hat{\gamma} = 5.0 \times 10^{-4} \text{ rad}(C) \right]$.

squares (NRSS):

$$\text{NRSS} = \sum_{i=1}^n [d_i - E(d_i)]^2 / E(d_i)$$

The d_i is the value of the observed random variable associated with the i th sample cumulative density point, and $E(d_i)$ is the value of the a priori random variable associated with the i th sample cumulative density point. No statistical goodness-of-fit test (probability statements) can be made with the NRSS because its distribution is unknown. However, the NRSS is a quantitative measure of the degree to which the sample data fit any specified distribution and can be used as a goodness-of-fit indicator.

Table IV summarizes the results obtained from applying the NRSS goodness-of-fit measure to the radiation-effects reliability data of Table I. As indicated in Table IV, this goodness-of-fit measure was applied to each of the failure distributions presented in Section II-1c.

(6) Selection Procedure

When using the preceding techniques to select the form of the unknown failure density function, the following recommendations can be made:

1. When possible, the parametric goodness-of-fit test should be used.
2. The first alternative is the use of the nonparametric goodness-of-fit test (in general, these tests are not as powerful as the parametric test).

3. When the parametric and nonparametric goodness-of-fit tests are not applicable, or when these tests are not powerful enough to discriminate between the a priori probability density functions, the selection of the underlying failure distribution should be based on (1) a graphical goodness-of-fit comparison based on the use of probability paper and (2) a comparison of the normalized residual sums of squares, with consideration given to other factors such as (a) the data fit at the left-hand tail of the distribution, where the fit is of most concern, and (b) analytical considerations.

Table V presents the conclusions that can be made, on the basis of the preceding goodness-of-fit analysis, with regard to the selection of a failure density function. Where possible, a rejection of the respective a priori failure distributions is stated. If a statistical rejection was not possible, a summary of the goodness-of-fit measures is listed. To verify the summary statements, refer back to the respective goodness-of-fit analyses.

Table V also illustrates a problem frequently encountered when dealing with statistical testing and inference based on the use of small sample sizes: that is, it is not possible to make definite acceptance or rejection statements in all cases. When this occurs, the problem of selecting the appropriate underlying failure distribution becomes somewhat subjective, and the various selection criteria must be weighted in direct proportion to their relative importance to the problem at hand.

In the work that follows (Section II-2c), the Weibull distribution is used as the form of the underlying failure distribution. By referring to Table V, it is seen that this

Table V
CONCLUSIONS ON THE SELECTION OF A FAILURE
DISTRIBUTION FOR THE 2N1613 TRANSISTORS

	Summary of the Goodness-of-Fit Test and/or Measures	
	-10% Change in h_{FE}	-20% Change in h_{FE}
A Priori Failure Distribution H_0		
Weibull $\beta > 1$	Minimum NRSS Best fit at the left tail of the distribution Nominal scatter about a good, approximating, straight line	Minimum NRSS Nominal fit at the left tail of the distribution Small scatter about a good, approximating, straight line
Exponential	Reject H_0 at the 90% level of significance	Reject H_0 at the 90% level of significance
Truncated Normal	Reject H_0 at the 90% level of significance	Maximum NRSS Nominal fit at the left tail of the distribution Nominal scatter about a poor, approximating, straight line
Log Normal	Reject H_0 at the 80% level of significance	Comparatively large NRSS Nominal fit at the left tail of the distribution Small scatter about a good, approximating, straight line
Extreme Value	Comparatively small NRSS Poor fit at the left tail of the distribution Nominal scatter about a poor, approximating, straight line	Comparatively small NRSS Nominal fit at the left tail of the distribution Nominal scatter about a good, approximating, straight line

distribution does not have any claim to being the statistically proven radiation-effects reliability failure distribution. However, strong evidence is presented as to this distribution's unique capability of expressing the observed relationship between the radiation parameter dose and reliability.

c. Estimation of the Weibull-Distribution Parameters

When the numerical values of the three Weibull parameters are known, it is quite easy to ascertain the properties of the specific distributions. However, when only sample data are available from which the values of the parameters are to be estimated, the problem becomes more difficult - difficult in the sense that because of the mathematical dependence between the parameters involved, there is no closed form for the independent estimation of each parameter. Many articles have been written on estimating the parameters of the Weibull distribution. However, for purposes of completeness the equations involved in the two best-known methods of estimation (i.e., moments and maximum likelihood) are presented below.

(1) Method of Moments

The method-of-moments procedure is to equate various sample properties with the corresponding population properties. Then, the population parameters to be estimated are solved for in terms of the known sample properties. The required population properties for the Weibull distribution are as follows:

$$\begin{aligned} \text{Mean} \quad \mu &= \gamma + a^{1/\beta} \Gamma(1 + 1/\beta) \\ \text{Variance} \quad \sigma^2 &= a^{2/\beta} \left[\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta) \right] \end{aligned}$$

$$\text{Skewness } \xi_3 = \frac{\Gamma(1 + 3/\beta) - 3\Gamma(1 + 2/\beta)\Gamma(1 + 1/\beta)}{[\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)]^{3/2}} + \frac{2\Gamma^3(1 + 1/\beta)}{[\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)]^{3/2}}$$

The three statistical measures of the sample data that are used to estimate μ , σ^2 , and ξ_3 are, respectively,

$$\begin{aligned}\bar{d} &= \sum_{i=1}^n d_i/n \\ s^2 &= \sum_{i=1}^n (d_i - \bar{d})^2/(n-1) \\ z_3 &= \sum_{i=1}^n [(d_i - \bar{d})^3/(n-1)] / s^3\end{aligned}$$

It is seen that it is quite difficult to obtain α , β , and γ explicitly in terms of the three measures of the data, \bar{d} , s^2 , and z_3 . However, given the ξ_3 estimate - that is, z_3 - an approximation of β can be obtained by solving the skewness equation graphically, since ξ_3 is a function only of the shape parameter β . With this estimate of β denoted as $\hat{\beta}$ and the sample value s^2 , an estimate of α is made:

$$\hat{\alpha} = \left\{ s^2 / \left[\Gamma(1 + 2/\hat{\beta}) - \Gamma^2(1 + 1/\hat{\beta}) \right] \right\}^{\hat{\beta}/2}$$

and, subsequently, an estimate of γ is made:

$$\hat{\gamma} = \bar{d} - \hat{\alpha}^{1/\hat{\beta}} \Gamma(1 + 1/\hat{\beta})$$

(2) Method of Maximum Likelihood

The equations of the maximum likelihood estimators, which

provide sufficient statistics if they exist, are:

$$\begin{aligned}
 & 1/\hat{\beta} + 1/n \sum_{i=1}^n \ln(d_i - \hat{\gamma}) \\
 & - \left[\sum (d_i - \hat{\gamma})^{\hat{\beta}} \ln(d_i - \hat{\gamma}) \right] \left[\sum (d_i - \hat{\gamma})^{\hat{\beta}} \right]^{-1} = 0 \\
 & (1 - \hat{\beta}) \sum (d_i - \hat{\gamma})^{-1} \\
 & + \left[n \sum (d_i - \hat{\gamma})^{\hat{\beta}-1} \right] \left[\sum (d_i - \hat{\gamma})^{\hat{\beta}} \right]^{-1} = 0
 \end{aligned}$$

There is no "closed form" solution to these two equations for $\hat{\beta}$ and $\hat{\gamma}$. A solution can be obtained by use of iterative techniques. Once these two estimates are obtained, the scale parameter α can be estimated:

$$\hat{\alpha} = \sum_{i=1}^n (d_i - \hat{\gamma})^{\hat{\beta}} / n$$

The numerical values obtained by use of the two methods differ somewhat; however, for small sample sizes there is nothing indicating which set of estimators is better. Application of statistical estimation theory indicates that the maximum likelihood estimators are better for large sample sizes - better in the sense that their variances approach a minimum attainable value as the sample size increases. For small sample sizes, as encountered in the radiation-effects reliability study, there is no obvious way of choosing between the two methods of estimation.

(3) Graphical Method

A third method of Weibull parameter estimation, which is presented by Kao (Ref. 6), is based on the use of graphical techniques. Graphical techniques are heavily employed in the data analysis of this study. Graphical techniques were selected for use in this study for the following reasons:

1. Large amounts of data can be quickly analyzed for general trends and parameter estimates.
2. When dealing with small sample sizes, there is no theoretical justification for selecting one estimation technique (that is, method of moments, method of maximum likelihood, or graphical method) over the others; however, an empirical analysis, outlined and summarized below, suggests that for nominal sample sizes the graphical method, which is based on the method of least squares, is at least as good as the other techniques.

(4) Comparison of the Weibull Parameter Estimators

As an illustration of the accuracy and difference in the numerical values obtained from each type of estimator, the following case problem is presented:

1. Sample data (N=100) were generated by Monte Carlo techniques from two Weibull cumulative density functions with known parameters.
2. Parameter estimates were obtained from the sample data using each of the estimation techniques.

The a priori distribution and the parameter estimates obtained by each method are presented in Table VI.

d. Weibull Data Analysis

By mathematical manipulation of the Weibull c.d.f.,

$$F(d) = 1 - \exp \left[- (d - \gamma)^B / \alpha \right]$$

one can arrive at an expression which readily lends itself to

Table VI

AN EMPIRICAL COMPARISON OF PARAMETER ESTIMATES FROM WEIBULL DATA^a

A Priori Distribution I		Method Used to Estimate Distribution Parameters				
Parameters	Values	Moments			Graphical	
		Case I ^b	Case II ^b	Maximum Likelihood	Case I	Case II
Location, γ	1.975	0.0	- 4.3335 ^c	0.0	0.0	1.3
Scale, α	166.827	304.11	644.997	599.195	171	165.8
Shape, β	1.3285	1.47	1.62	1.6308	1.41	1.35

A Priori Distribution II		Location Parameter Known to be Zero				
Parameters	Values	Moments			Graphical	
		Case I	Case II	Maximum Likelihood	Case I	Case II
Location, γ	0.0	0.0	0.0	0.0	0.0	0.0
Scale, α	166.827	240.20	210.777	161	161	161
Shape, β	1.3285	1.435	1.2456	1.316	1.316	1.316

^aThe sample data and parameter estimates, with the exception of the graphical estimates, were taken from Reference 7. The sample size $N = 100$.

^bCase I: Location parameter unknown but assumed equal to zero.

Case II: Location parameter unknown and assumed not equal to zero.

^cAn inadmissible estimate, since location parameter must be greater than zero.

graphical parameter estimation, as shown below.

Consider the ratio

$$1/[1 - F(d)] = \exp(d^\beta / \alpha), \quad \gamma = 0$$

By taking the natural logarithm of this ratio twice,

$$\ln \ln \{1/[1 - F(d)]\} = \beta \ln d - \ln \alpha$$

an equation which has a linear form is obtained, namely,

$$Y = mX + c$$

where

$$Y = \ln \ln \{1/[1 - F(d)]\}$$

$$m = \beta$$

$$X = \ln d$$

$$c = -\ln \alpha$$

Thus, on Weibull probability paper, which has an ordinate scale of $\ln \ln \{1/[1 - F(d)]\}$ and an abscissa scale of $\ln d$, the c.d.f. of a Weibull distribution will plot as a straight line for which the y intercept and the slope are estimates of $-\ln \alpha$ and β , respectively.

Weibull probability paper also has an auxiliary coordinate system. The auxiliary scales are nonlinear and are to be used in the direct plotting of the raw data. The abscissa and ordinate scales of the two coordinate systems are proportioned to one another in such a manner that the probability paper converts the relationships expressed on the nonlinear raw-data coordinate

system into the identical relationship on the linear Weibull coordinate system. If upon the Weibull coordinate system the relationship takes the linear form $Y = mX + c$ mentioned above, then estimates of α and β can be taken from the graph paper.

An illustration of using Weibull probability paper for parameter estimation is presented below. The failure data given in Table VII and plotted in Figure 12 are taken from a set of 2N718¹ transistors tested in a radiation environment.

Table VII

2N718A TRANSISTORS TESTED IN A RADIATION
ENVIRONMENT^a

Rank i	Failure Dose $d \times 10^{-4}$	Percent Failure $F(d) = (\frac{i}{n+1})$	Rank i	Failure Dose $d \times 10^{-4}$	Percent Failure $F(d) = (\frac{i}{n+1})$
1	1.39	0.091	5	3.60	0.455
2	2.48	0.182	7.5	4.30	0.637
3	2.58	0.273	7.5	4.40	0.728
5	3.50	0.455	9	5.03	0.819
5	3.55	0.455	10	5.76	0.910

^a The data are from the NARF-LMSC test for a failure criteria of a -10% change in h_{FE} with $V_{CE} = 20$ v and $I_C = -10$ ma. The sample size was 10.

In plotting the sample c.d.f., $1/(n+1)$ is used for the ith sample cumulative density point to estimate $F(d_i)$ (see Fig. 12).

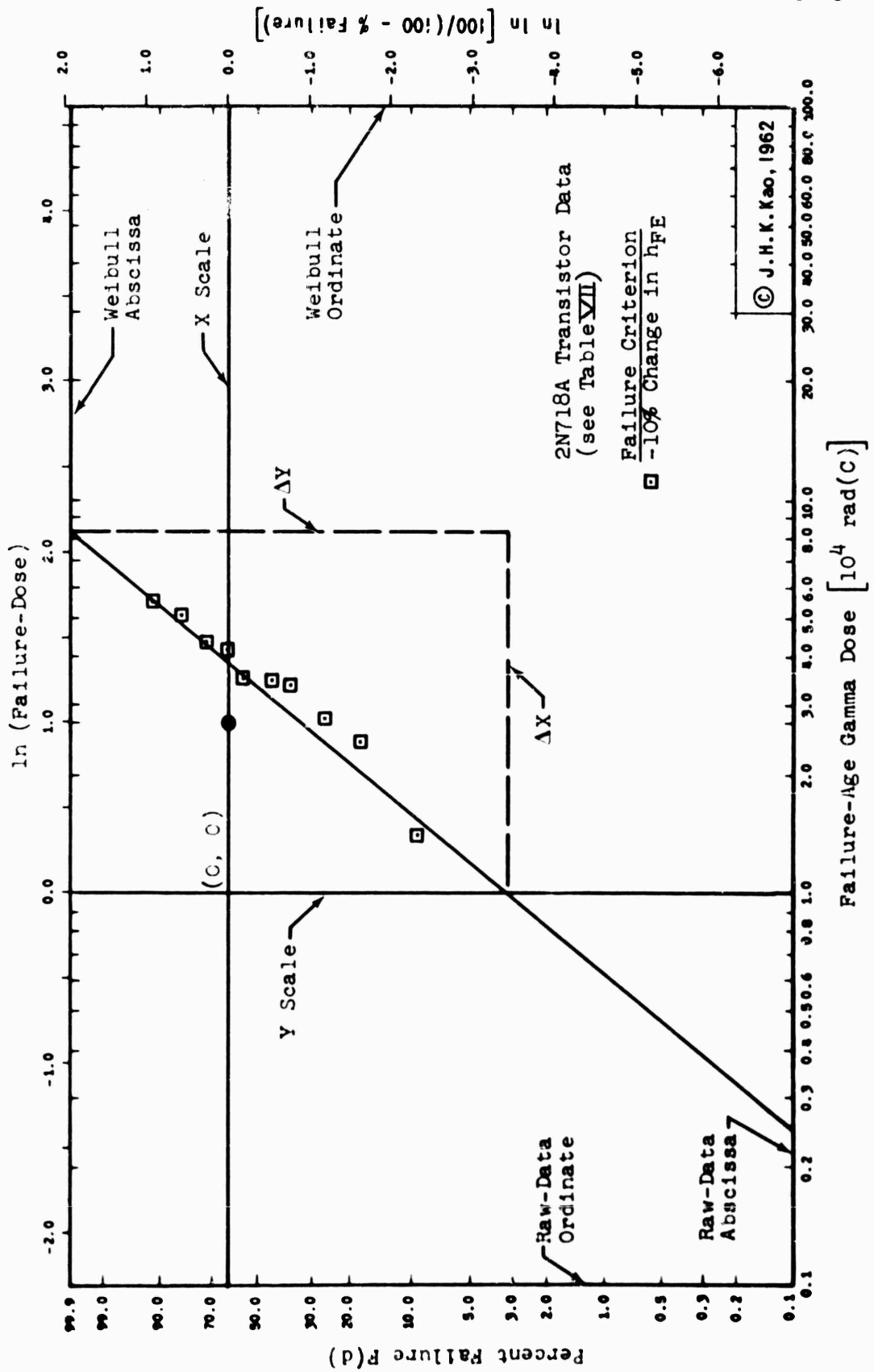


Figure 12 Parameter Estimation from Weibull Probability
Paper: $\gamma = 0$

The transfer is made from the raw data scales d and $F(\hat{d})$ to the Weibull scales $\ln(d)$ and $\ln \ln \left\{ 1 / [1 - F(d)] \right\}$, respectively. On the Weibull scales the slope of the c.d.f. data points and the y intercept are the estimates of β and $-\ln \alpha$, respectively. For the 2N718A transistor data tabulated in Table VII and presented in Figure 12,

$$\begin{array}{lll} \text{y intercept} & = & -3.4 \\ -\ln \hat{\alpha} & = & -3.4 \\ \hat{\alpha} & = & 30.0 \end{array} \qquad \begin{array}{l} \frac{y}{x} = \frac{5.40}{2.16} \\ \hat{\beta} = 2.50 \end{array}$$

Thus, the estimated Weibull density function is

$$F(\hat{d}) = 1 - \exp \left[-(d)^{2.5}/30.0 \right].$$

The reliability for any gamma dose, d , can be obtained by direct substitution into the scaled reliability model

$$R(\hat{d}) = \exp \left[-(d)^{2.5}/30.0 \right].$$

As indicated in Table VII and Figure 12, the parameter estimates of this model are based on a scale in which 1 unit = 1×10^4 rad(C). The reliability model may be adjusted for use on any scale desired by applying a linear transformation, as illustrated in a later paragraph.

(1) The Sample Cumulative Density Function

There are several different estimators that are commonly used in data analysis to estimate the population c.d.f., $F(d_1)$. These estimators, together with a brief discussion of their properties, are presented below.

From a sample of size n , let $d_1 < d_2 < \dots < d_n$ be the radiation doses at which the failures occur. Let $F(\hat{d}_1)$ be the sample estimate of $F(d_1)$, the true percent of the population elements that fail on or before dose d_1 . An unbiased estimator of $F(d_1)$ is its sample mean, $i/(n+1)$, where i is the number of sample elements that have failed on or before dose d_1 . Thus,

Case I. $F(\hat{d}_1) = i/(n+1)$

The sample c.d.f. - namely, the sample proportion failing on or before dose d_1 - is another frequently used c.d.f. estimator. Thus,

Case II. $F(\hat{d}_1) = i/n$

It is a common practice to use the Case I and Case II estimators for small and large sample sizes, respectively. If the point of changing estimators is taken to be a sample size of $n = 25$, the maximum difference which can occur between $F(\hat{d}_1)$ Case I and $F(\hat{d}_1)$ Case II is $4/100$, which is negligible when applied to a graphical plot.

In the Case II estimator, $F(\hat{d}_1)$ increases by increments of $(1/n)$. A frequently used variation of this step-function is given by

Case III. $F(\hat{d}_1) = (i - 1/2)/n$

This estimator is simply the midpoint of each incremental change of the Case II estimator.

Another estimator that is occasionally employed is derived from the method of median ranks. The median rank is an estimate of the cumulative percent failure for each ordered failure such

that, in the long run, the positive and negative errors of the estimates cancel each other. Tables of median ranks are presented in Reference 8. However, the following equation, developed by A. Benard in 1953, is an approximation that can be used to estimate the i th median rank, where median rank $\approx (i - 0.3)/(n + 0.4)$; hence,

Case IV. $F(\hat{d}_i) = (i - 0.3)/(n + 0.4)$

The Case I estimator, which is an unbiased estimator, is recommended.

(2) Tied Observations

When ties or indistinguishable differences occur among the failure doses, as was the case in Table VII, the following procedure can be used:

1. The median rank is assigned to each of the tied values.
2. Only one percentage failure point, $F(d_i)$, is plotted for each group of ties, that being the mean percentage failure of the tied values.

However, it should be noted that if the least-squares line is to be calculated analytically rather than estimated graphically, each percentage failure point, $F(d_i)$, is to be treated individually, regardless of whether or not tied observations occur.

(3) Treatment of the Location Parameter.

In the previous example of Weibull parameter estimation by graphical techniques, the location parameter, γ , was assumed to be zero. Employing the procedure outlined in the paragraph discussing the general approach, without the $\gamma = 0$ assumption, results in

$$\ln \ln(1/[1 - F(d)]) = B \ln(d - \gamma) - \ln a$$

Thus, for a Weibull distribution in which $\gamma \neq 0$, the percent of failures, $F(d)$, must be plotted against the failure dose minus gamma, $(d - \gamma)$, in order to use the Weibull scales as before to estimate α and β .

Consequently, an estimate of γ (the zero assumption or otherwise) must be made before estimates of the Weibull parameters can be obtained graphically. There are three approaches, any one of which may be used to estimate the location parameter. The three different approaches are outlined below as Cases I, II, and III.

Case I

1. Plot the data points
2. Draw the approximating straight line
3. Assume $\gamma = 0$
4. Obtain the α and β estimates from the approximating straight line

Case II

1. Step 1 of Case I
2. Fit a curve through the data points (linear or curvilinear); let γ = the 0.1% failure intercept
3. Replot $F(d)$ vs $(d - \hat{\gamma})$
4. Draw a new approximating straight line
5. Obtain the α and β estimates from the new approximating straight line

Case III

1. Steps 1, 2, 3, and 4 of Case II
2. When both approaches (Cases I and II) fail to produce a linear plot, values within the range of 0 to the 0.1% failure intercept are successively chosen as $\hat{\gamma}$ until linearity is produced for the plot of $F(d)$ vs $(d - \hat{\gamma})$

3. Step 5 of Case II

A situation in which $\gamma \neq 0$ is illustrated in Figure 13. The failure data from which Figure 13 was constructed are tabulated in Table I. As seen in Figure 13, the original data produce a curvilinear plot. Consequently, Case I ($\gamma = 0$) is not applicable. Following the approach outlined in Case II, the 0.1% failure intercept and estimate of gamma is seen to be $\hat{\gamma} = 5.0$ units. Step 3, a replot of $F(\hat{d}_1)$ vs $(d_1 - \hat{\gamma})$, is needed to see whether linearity has been produced; if it has been produced, one proceeds with Steps 4 and 5. As illustrated in Figure 13, the estimate $\hat{\gamma} = 5.0$ units in the transformed data approximates a linear plot; thus, proceeding with Steps 4 and 5 yields

$$\begin{aligned} -\ln \hat{\alpha} &= -0.80 & \Delta y / \Delta x &= 2.72/2.30 \\ \hat{\alpha} &= 2.23 & \hat{\beta} &= 1.18 \end{aligned}$$

and the estimated Weibull distribution function is

$$F(\hat{d}) = 1 - \exp \left[- (d - 5.0)^{1.18} / 2.23 \right], \quad d \geq 5.0$$

It is a common practice in reliability work to define the location parameter γ as a "guarantee" period within which no failure can occur. This, of course, implies that γ is some number greater than zero. However, it should be noted that the addition of any constant to the raw data can be a useful technique for obtaining a linear plot. That is, just because a negative γ does not fit into the guarantee period concept, it should not be discarded as a legitimate transformation for producing

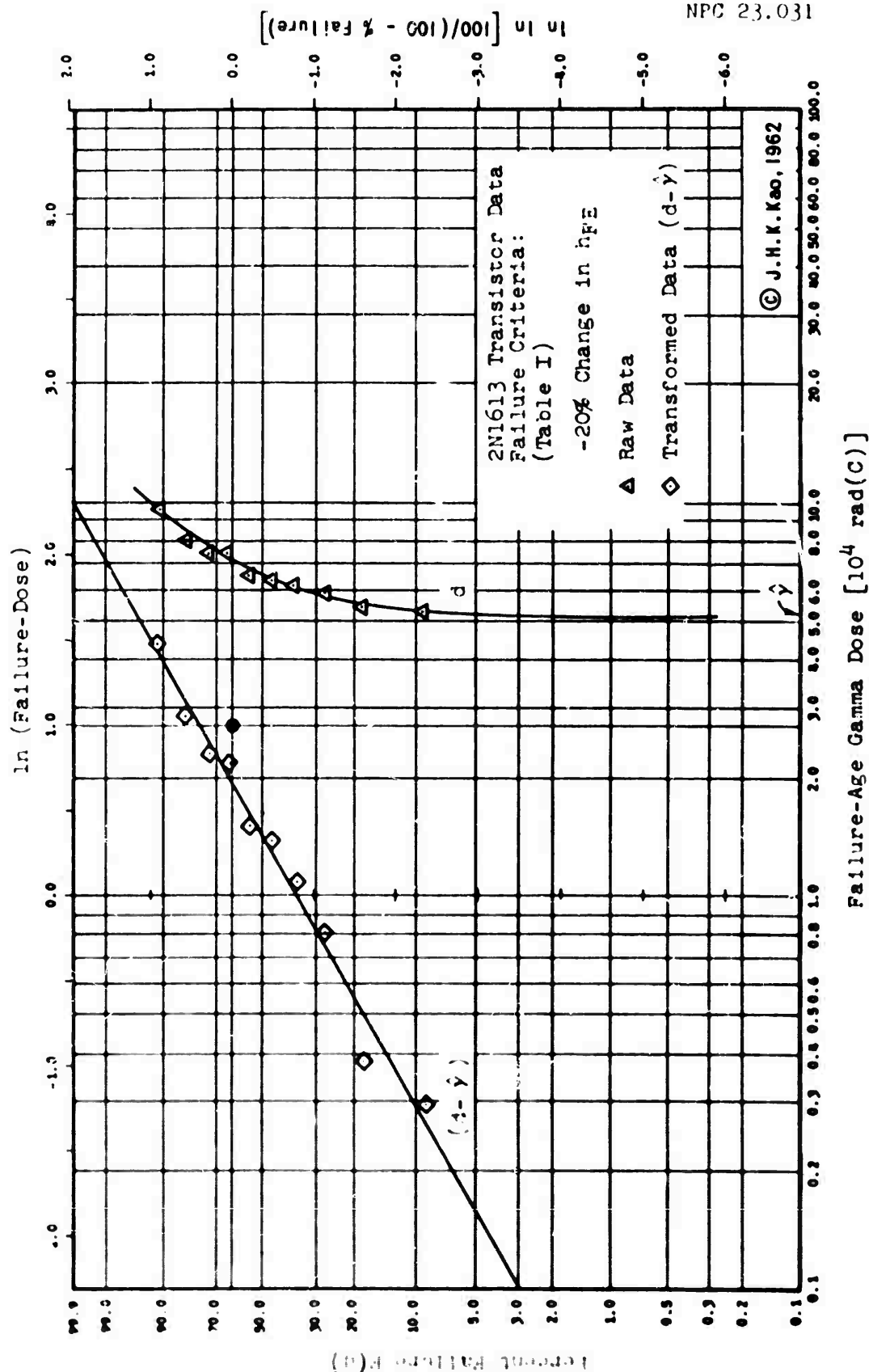


Figure 13 Parameter Estimation from Weibull Probability
Paper: $\gamma > 0$

a linear plot on Weibull probability paper. As long as the final approximating line from which the parameter estimates are taken is linear, the interpretation of γ is primarily a matter of philosophy. An example of such a transformation, with $\hat{\gamma} = -1.5$, is presented in Figure 14.

(4) Change of Scale

A procedure commonly used to facilitate data analysis is that of changing scales. For example, in the problem of parameter estimation it is often advantageous to apply a linear transformation to the raw data before beginning the task of estimating the values of the population parameters from the sample data. The procedure is to apply the desired linear transformation to the raw data, determine the estimates of the parameters on the transformed scale, then determine the relationship that will convert these estimates back to the original scale.

The effect of a linear transformation upon the Weibull distribution is illustrated below. Consider the Weibull p.d.f. for which $\gamma = 0$:

$$f(d) = \left[\beta d^{\beta-1} / \alpha \right] \exp(-d^\beta / \alpha)$$

If a linear transformation $y = cd$ is made, where c is a constant, the distribution of y is

$$f(y) = \left[\beta y^{\beta-1} / \alpha c \right] \exp \left[-y^\beta / \alpha c^\beta \right]$$

In the preceding 2N718A transistor example, the parameter estimates were based on a scale in which 1 unit equals 10^4 rad(C), and the resulting p.d.f. was

$$f(d) = (2.5d^{1.5}/30.0)\exp(-d^{2.5}/30.0)$$

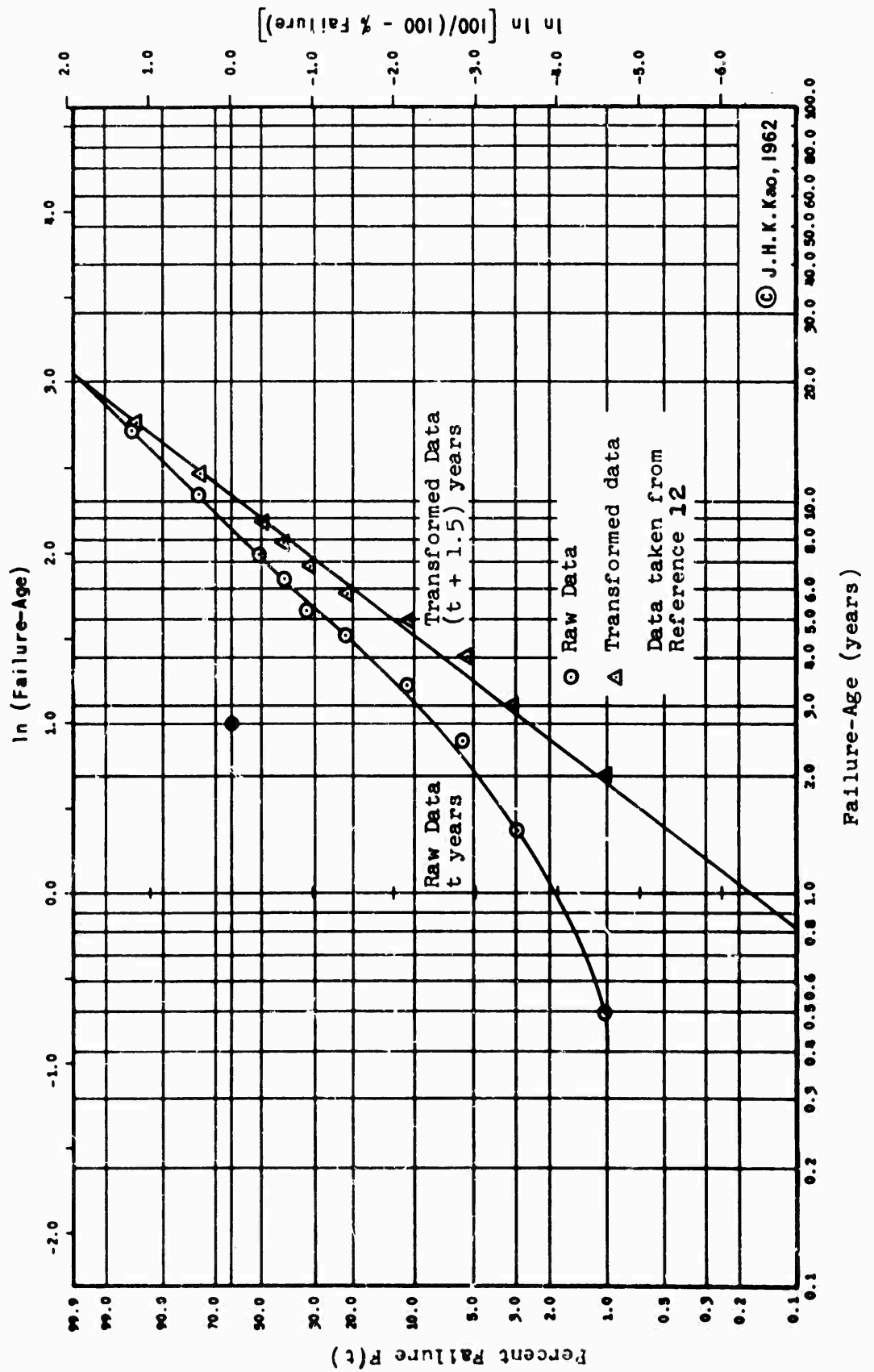


Figure 14 Parameter Estimation from Weibull Probability
Paper: $\gamma < 0$

To convert the parameter estimates and the resulting distribution to a scale in which 1 unit equals 1 rad(C), the following inverse linear transformation is applied:

$$y = cd$$

$$y = 10^4 d$$

then

$$f(y) = (2.5y^{1.5}/30.0 \times 10^{10}) \exp(-y^{2.5}/30.0 \times 10^{10})$$

where 1 unit on the y scale equals 1 rad(C).

(5) Determination of the Maximum Allowable Dose

Once the parameters of the reliability distribution have been estimated from radiation-effects reliability test data, the following question may arise: What is the maximum gamma dose to which a transistor may be subjected and still maintain a specified reliability level? The techniques involved in determining this critical dose, d_c , depends upon the level of reliability specified.

Case I. If the reliability level in question is within the range 0.001 - 0.999, the critical dose can be read directly from the Weibull probability paper.

For example, the critical dose for a 0.99 reliability is the failure dose which corresponds to the 1.0% failure point. Thus, for the 2N718A transistor example (see Fig. 15), the critical dose for a 99% reliability is 0.60×10^4 rad(C). Consequently, it can be concluded that the probability is 0.99 that a 2N718A transistor will not fail because of radiation effects, as long as it does not receive a gamma dose equal to or greater than 6,000 rad(C).

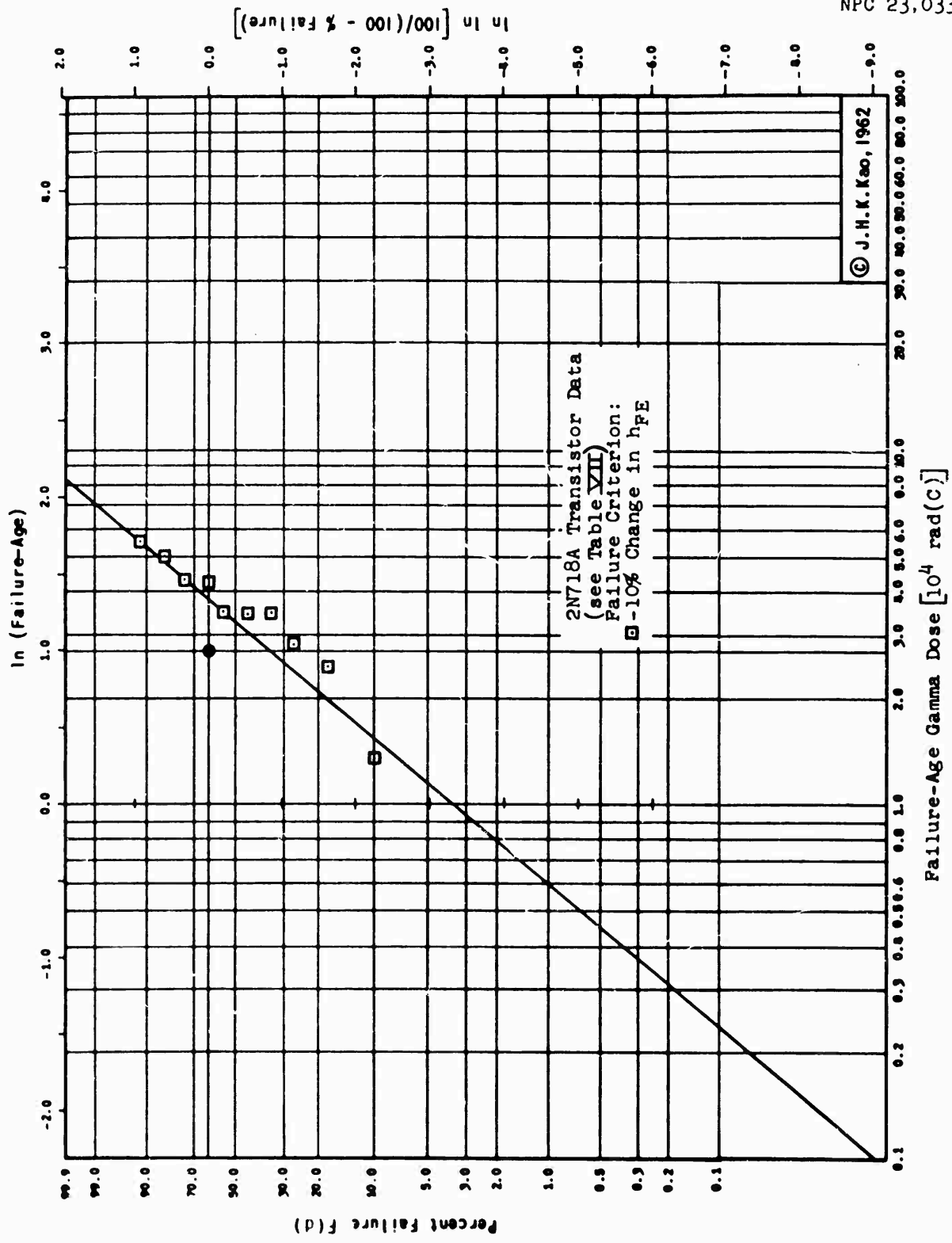


Figure 15 Weibull Probability Paper with an Extreme Extended Scale

Case II. If the reliability level in question is greater than 0.999, the critical dose can be determined by either of the following two methods.

Extension of the Weibull Probability Paper:

For reliabilities greater than 0.999, an extension of the Weibull probability paper can be made so that a graphical determination of the critical dose can be obtained as described in Case I. For example, by extending the percent failure scale one cycle lower (from 0.001 to 0.0001), the maximum allowable dose corresponding to a 0.9999 reliability is given by the intersection of the least-squares line and the 0.01% failure plane. Figure 15 illustrates the extension of the probability paper for the data of the 2N718A transistor example. The 0.01% failure intercept occurs off of the scale; however, by extrapolation, an estimate of 0.099×10^4 rad(C) is made.

Analytical Approximation: The Weibull c.d.f. can be expressed as

$$F(d) = (d - \gamma)^\beta / \alpha + o(d - \gamma)^\beta / \alpha$$

More specifically, for reliabilities > 0.999

$$F(d) = (d - \gamma)^\beta / \alpha + \epsilon, \quad \epsilon < 0.5 \times 10^{-6}$$

Consequently, the maximum dose to which the transistor may be exposed and maintain a reliability greater than 0.999 can be approximated by the following equation

$$d_c = \left\{ [1 - R(d)] \alpha \right\}^{1/\beta} + \gamma$$

where $R(d)$ is the desired reliability. For the 2N718A transistor example of Figure 15, the parameter estimates are $\gamma = 0$, $\alpha = 30$, and $\beta = 2.5$. Application of the analytical approximation technique to determine the maximum dose to which the 2N718A transistor can be subjected and still declare a 0.9999 reliability results in

$$d_c = [(0.0001)(30.0)]^{1/2.5}$$

$$\log_{10}(d_c) = 8.9908 - 10$$

$$d_c = 0.0996 \times 10^4 \text{ rad(C)}$$

Note that the solution for d by means of the analytical approximation^c verifies the result obtained from the extension of the Weibull graph paper.

(6) Confidence Intervals

When estimating a failure distribution from sample data, it is frequently not sufficient to simply express the hypothesized c.d.f. in terms of its estimated parameters. Instead, it is more meaningful to determine an interval that will have some specified probability of including the true c.d.f. The boundary values of such an interval are called the confidence limits of the c.d.f., while the interval itself is called the confidence interval for the c.d.f. The confidence coefficient is the relative frequency with which the confidence interval will contain the true c.d.f. (in the sense that if many estimates of the c.d.f. are made, the corresponding confidence intervals associated with these estimates will contain the true c.d.f. in a proportion of times equal to the value of the confidence coefficient). Thus, a $(1 - \eta)\%$ confidence interval for the c.d.f. means that, on the average, the confidence intervals associated with many estimates of the c.d.f. will encompass the true value of the c.d.f. $(1 - \eta)\%$ of the time.

Just as point-by-point estimates have been made for the Weibull c.d.f., it is also possible to construct an interval-by-interval confidence band. If the transformation $Y_1 = F(d_1)$

is applied to a set of ordered observations $d_1 < d_2 < \dots < d_n$ that are randomly drawn from a single density function, then it can be shown that the p.d.f. of Y_1 is the following beta distribution:

$$h(y_1) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} y_1^{p-1} (1-y_1)^{q-1}$$

where $p = 1$, $q = n + 1 - i$, and $0 < y_1 < 1$.

The c.d.f. of Y_1 , denoted by $H(y_1)$, is the incomplete beta function

$$H(y_1) = I_{y_1}(p, q) = \int_0^{y_1} h(x) dx$$

Consequently, by solving $I_{y_1}(p, q)$ for various percentiles, it is possible to obtain interval estimates for $F(d_1)$, $i = 1, 2, \dots, n$. More specifically, the upper and lower confidence limits for a confidence coefficient of $100(1 - \eta)\%$ are, respectively (see Ref. 6):

$$U(y_1) = H_{y_1}^{-1}(1 - \eta/2)$$

$$L(y_1) = H_{y_1}^{-1}(\eta/2)$$

Tables of the incomplete beta function are required for solving $I_{y_1}(p, q)$. The incomplete beta function tables contain only those values of $I_y(p, q)$ for which p is equal to or greater than q . If a value of $I_y(p, q)$ is required in which p is less than q , the following relationship can be employed:

$$I_y(p, q) = 1 - I_{1-y}(q, p)$$

The failure data tabulated in Table VII and presented in

Figure 12 are used to illustrate the construction and use of both 1- and 2-sided confidence intervals. The working data pertinent to the construction of the confidence intervals are tabulated in Table VIII. To illustrate the techniques involved in calculating the elements of Table VIII, elements in row 8, column 2 and row 3, column 1 will be evaluated.

Table VIII
CONFIDENCE INTERVALS FOR THE 2N718A TRANSISTOR DATA

Rank i	90% Two-Sided $L(y_i)$ $U(y_i)$		80% Two-Sided $L(y_i)$ $U(y_i)$		80% One-Sided $L(y_i)$ $U(y_i)$	
1	0.005	0.206	0.010	0.206	0.022	0.149
2	0.036	0.39	0.054	0.34	0.083	0.270
3	0.087	0.51	0.115	0.45	0.158	0.38
4	0.150	0.61	0.177	0.55	0.239	0.48
5	0.222	0.70	0.267	0.65	0.33	0.58
6	0.30	0.78	0.35	0.73	0.42	0.67
7	0.39	0.85	0.45	0.82	0.52	0.76
8	0.49	0.91	0.55	0.89	0.62	0.84
9	0.61	0.96	0.66	0.95	0.73	0.92
10	0.74	1.00	0.72	0.99	0.85	0.98

For Row 8, Column 2, the lower limit of the 2-sided 80%

confidence interval for $F(d_8)$ is:

$$L(y_8) = H_{y_8}^{-1}(0.10) = I_{y_8}^{-1}(8,3)$$

$$p = i = 8, \quad q = n + 1 - i = 3$$

and may be solved for by using a table of incomplete beta functions. In this case, Table I of Reference 9 gives the solution of $I_{y_8}(8,3)$ at the 0.10 level to be $y_8 = 0.55$, as shown in Table VIII. For row 3, column 1, the upper limit of the 2-sided 90% confidence interval for $F(d_3)$ is

$$U(y_3) = H_{y_3}^{-1}(0.95) = I_{y_3}^{-1}(3,8)$$

$$p = i = 3, \quad q = n + 1 - i = 8$$

This expression is not in Reference 9 per se; however, it may be solved by using the previously stated identity,

$$I_{y_3}(3,8) = 1 - I_{1-y_3}(8,3).$$

Table I of Reference 9 gives the solution of $I_{1-y_3}(8,3)$ at the 0.95 level to be $(1 - y_3) = 0.493$ or $y_3 = 0.507$, as shown in Table VIII.

Figure 16 illustrates the 2-sided confidence intervals. Note that these intervals extend from the points $F(d_1)$ equals $1/(n + 1)$ to $n/(n + 1)$; in this case, with a sample size of 10, the range is from 0.09 to 0.91. Within this range, the confidence intervals may be used to make the following type of

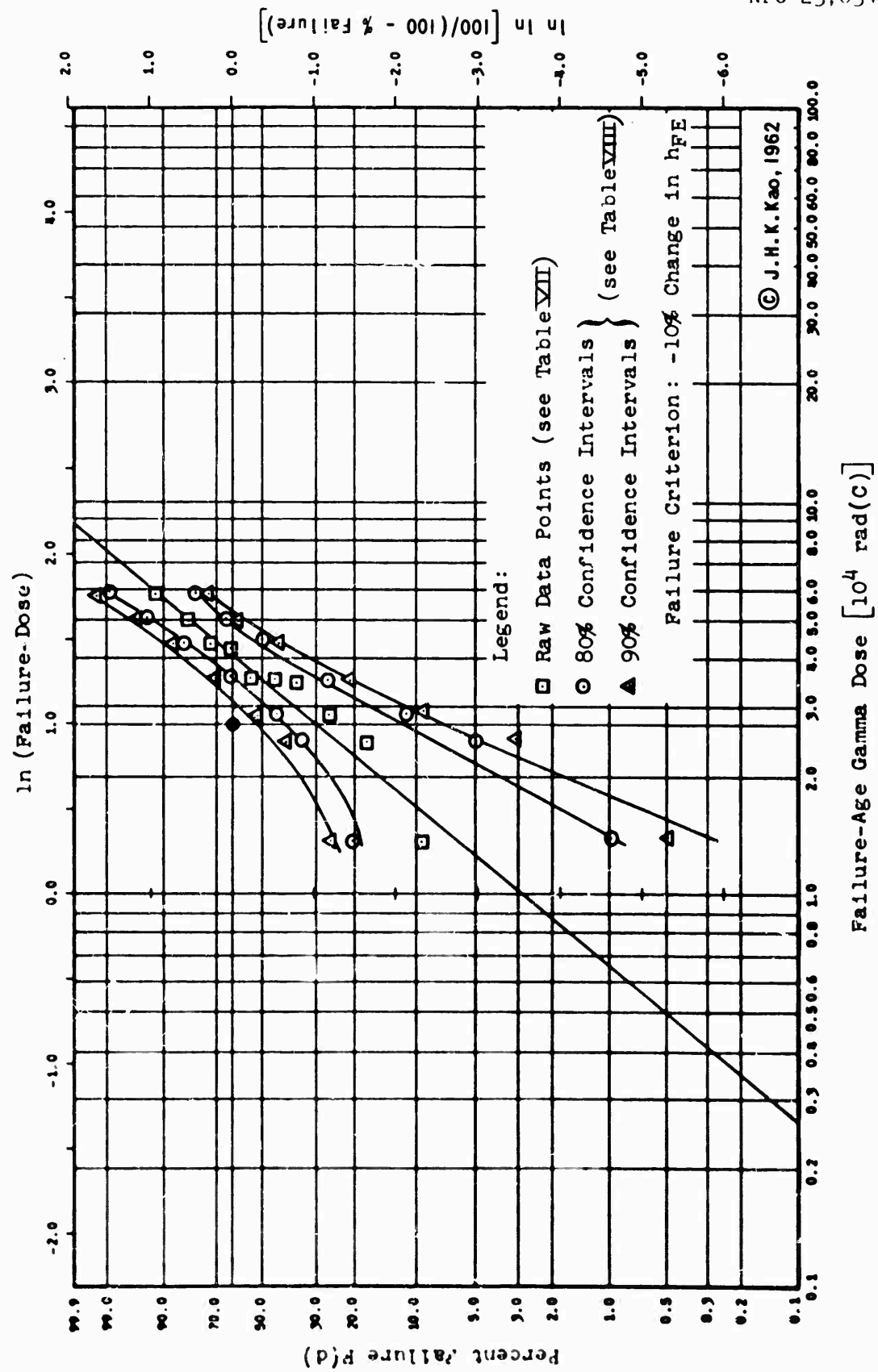


Figure 16 Two-Sided Confidence Intervals for the 2N718A Transistor Data

confidence statement:

$$0.80 \leq R[d = 1.5 \times 10^4 \text{ rad(C)}] \leq 0.99$$

That is, it can be stated with 80% confidence that the probability is between 0.80 and 0.99 that a 2N718A transistor will not fail because of radiation effects when subjected to a gamma dose of 1.5×10^4 rad(C). In most cases, it is not the reliability interval that is of primary interest; instead, it is the lower limit, beyond which the reliability is greater than some specified value for a prestated confidence level. This leads to the consideration of 1-sided confidence intervals, as illustrated in Figure 17. Also, the 1-sided confidence intervals will allow for a somewhat greater reliability statement at the same confidence level. For example, by using the 1-sided confidence intervals, it can be stated with 80% confidence that the radiation reliability of a 2N718A transistor is at least 0.87 if it is not subjected to a total gamma dose of more than 1.5×10^4 rad(C).

For reliabilities greater than $1 - 1/(n + 1)$, the temptation arises to continue (that is, extrapolate) the confidence limits, as illustrated by the dashed lines in Figure 17. However, it should be recognized that the accuracy achieved by extrapolation of any experimentally determined function beyond the range over which it was determined is highly questionable.

e. Estimation of Failure-Rate Functions

This subsection presents the techniques and problems associated with two different approaches to the estimation of failure-

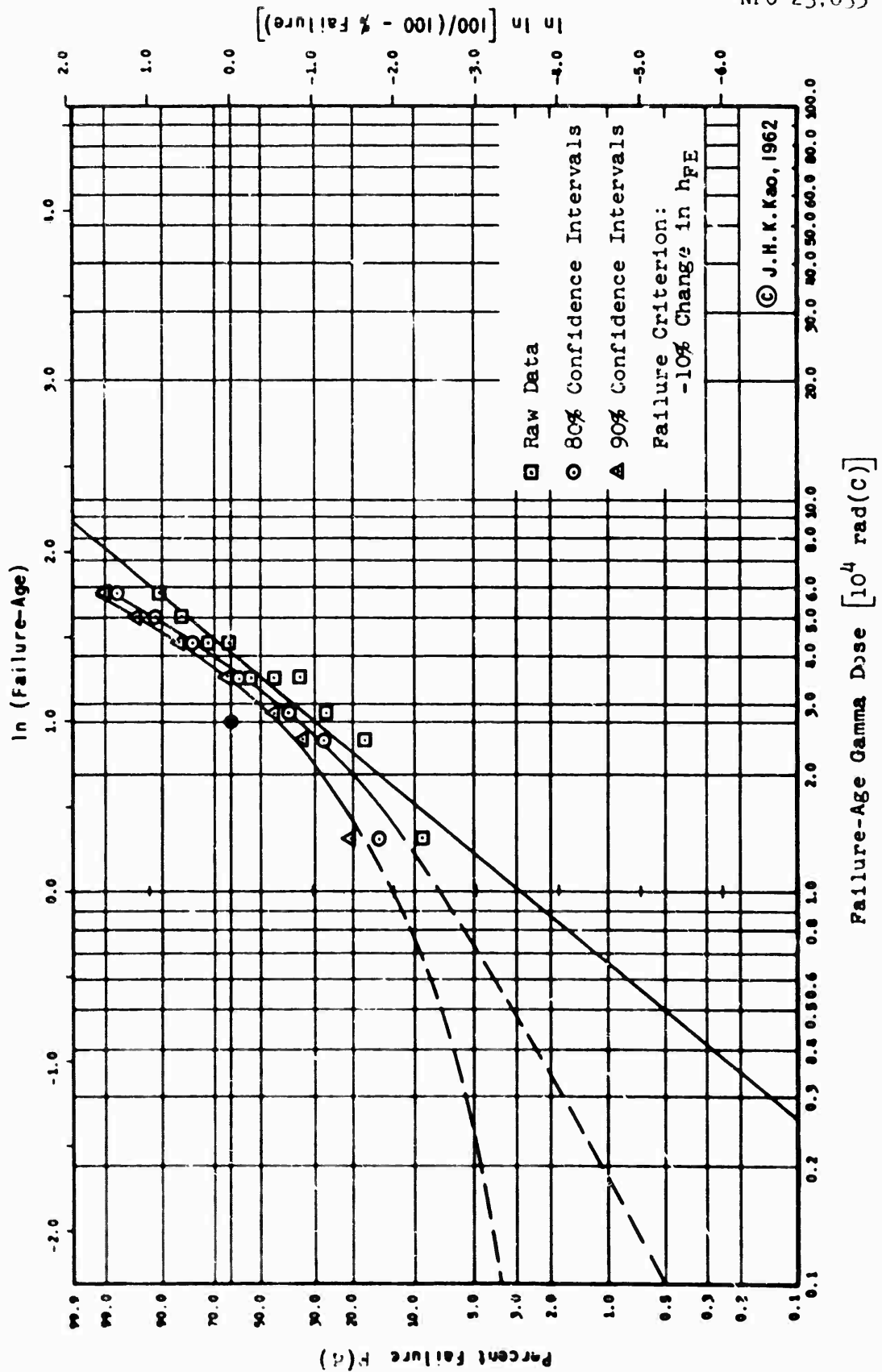


Figure 17 One-Sided Confidence Intervals for the 2N718A Transistor Data

rate functions. These two approaches involve (1) the previously considered failure-rate model and (2) a failure-rate multiple-regression analysis.

(1) Factorial Experiments in Radiation Testing

The procedure employed for estimating the failure-rate function $g(t)$, discussed in Section II-1b, is the determining factor of this model's capabilities. If the required set of factorial experiments is performed so that the main effects (chance and radiation rate, ranging over the spectrum of interest) of each factor (chance, radiation rate, and the effects of the interaction) can be determined, this model can be very useful (Ref. 10).

A common procedure in radiation life-testing is to accelerate the failure of components by testing at an extreme radiation dose for a short period of time and then attempting to extrapolate the accelerated test data back to what would be the regularly encountered operating conditions. If the criterion for accelerated testing can be met - that is, the interaction effect $I(t_C, t_R)$ is zero - the model is additive. This is equivalent to saying that the effects of radiation-induced failures can be evaluated independently of the effects of chance failures, and vice versa. This would be a highly desirable property, for radiation effects and reliability experiments have not been conducted together in any large experimental program. Consequently, most of the data available fall into one of two categories: (1) radiation effects (reliability not considered) or (2) reliability (radiation effects not considered). These data consist of experimentally determined lifetime distributions for similar

components operating in a normal environment and in a radiation environment. Because of the high dose rate and the short time span upon which the irradiation lifetime distributions are based, the contribution of time or chance failures to the failure distribution is most generally assumed to be insignificant. Therefore, the empirical failure distributions are a function of either time or radiation effects, but not both. Assuming the additive property can be demonstrated, the failure-rate model becomes

$$F(t) = 1 - \exp \left\{ - \int_0^t [g_1(x;C) + g_2(x;R)] dx \right\}$$

where the failure rate components $g_1(t;C)$ and $g_2(t;R)$ are determined independently of one another. The general procedure followed in this case is outlined below.

The failure distribution of the equipment operating in a normal environment and consequently subject to chance failures only is determined; that is, the failure rate of the following model is determined:

$$F(t) = 1 - \exp \left[- \int_0^t g_1(x;C) dx \right] \quad (\text{Chance failures only})$$

Similarly, the failure distribution of the equipment is determined when it is subject to radiation failures only, i.e., the failure rate of the following model is determined:

$$F(d) = 1 - \exp \left[- \int_0^d g_2(x;R) dx \right] \quad (\text{Radiation failures only})$$

Suppose, on the basis of the independent experiments, the exponent distribution ($\beta = 1$) and the Weibull distribution ($\beta > 1$) are

found to be appropriate for representing the chance and radiation-effects failures, respectively. The independently determined failure rates are then

$$g_1(t) = 1/\alpha_1$$

$$g_2(d) = \beta(d - \gamma)^{\beta-1} / \alpha_2$$

where t and d represent the variables time and total dose, respectively. To express both failure rates as a function of time only, the parameter average radiation rate, \bar{r} , is introduced and $g_2(d)$ becomes

$$g_2(t) = \beta(\bar{r}t - \gamma)^{\beta-1} / \alpha_2$$

By combining the two failure rates and integrating the exponent, the combined failure distribution is expressed as a function of time only, with the average radiation rate included as a parameter:

$$F(t) = 1 - \exp \left[-t/\alpha_1 - (\bar{r}t - \gamma)^{\beta} / \alpha_2 \right] .$$

Once again this approach is applicable only when the failure forces are additive, i.e., independent of one another.

(2) Failure Rate Multiple Regression Analysis

To date, practically all of the studies in the radiation-effects reliability area have followed the general approach outlined below.

1. Various probability density functions are postulated (on the basis of logical deduction, empirical goodness of fit, and ease of analytical considerations) as being the underlying failure distribution.
2. Once a specific failure distribution is postulated,

the problem is reduced to one of estimating the parameters of that distribution.

This procedure is, of course, only as good as the accuracy of the postulated failure distribution. No amount of effort expended in the parameter estimation phase can improve the accuracy of the final results if the hypothesized failure distribution is in error. Ironical as it may be, among the existing studies on radiation reliability, the parameter estimation phase is treated more diligently than the distribution selection phase.

In some situations it is reasonable to postulate a simple mathematical form of the failure distribution and consequently reduce the problem to that of estimating its parameters. However, this requires either a special knowledge of the physical properties of the materials involved or extensive empirical data. As one possible means of analyzing the radiation effects reliability data, the following failure-rate multiple-regression technique is introduced.

Consider the reliability model

$$R(d) = \exp \left[- \int_0^d g(x) dx \right], \quad d \geq 0 \quad (6)$$

where $g(d)$ is any function such that

$$R(0) = 1 \quad (7a)$$

$$R(\infty) = 0 \quad (7b)$$

$$R'(d) \leq 0, \quad d \geq 0 \quad (7c)$$

By integrating the exponent, it is seen that the reliability

function may be expressed in terms of the failure-rate function as

$$R(d) = e^{-y(d)} \quad (8)$$

where $y(d)$ is the integral of the failure rate $g(d)$.

The failure-rate regression approach defined herein does not specify the exact form of the underlying failure distribution. It only assumes (1) that there exists a transformation, $y(d)$, which transforms the random variable failure dose into a random variable that is exponentially distributed and (2) that transformation $y(d)$ can be approximated by a polynomial of relatively low degree.

From Equations 7a, 7b, and 7c, it is seen that the following conditions are imposed upon the polynomial $y(d)$:

$$y(0) = 0 \quad (9a)$$

$$y(\infty) = \infty \quad (9b)$$

$$y'(d) \geq 0, \quad d \geq 0. \quad (9c)$$

Taking natural logarithms of the reliability model, Equation 8, and approximating $y(d)$ with a polynomial of degree j results in

$$-\ln R(d) = b_0 + b_1 d + b_2 d^2 + \dots + b_j d^j. \quad (10)$$

The problem is now that of determining the appropriate degree j and the coefficients b_i ($i = 0, 1, \dots, j$) of the polynomial in Equation 10. These unknowns are to be determined by means of a multiple-regression analysis based on radiation-effects reliability test data. Estimates of $R(d)$ are obtained by taking

$R(d)$ to be 1 minus the percent of sample elements failing on or before dose d , i.e.,

$$R(\hat{d}_1) = 1 - F(\hat{d}_1)$$

The technique defined herein for determination of $y(d)$ is to initially impose only condition 9a upon the polynomial, apply standard regression techniques, and accept only those solutions for which conditions 9b and 9c are also satisfied. Condition 9a, $y(0) = 0$, implies that $b_0 = 0$. Thus,

$$y(d) = b_1d + b_2d^2 + \dots + b_jd^j$$

and the determination of $y(d)$ requires the fitting of a multiple-regression equation that is forced through the origin.

Consequently, the reliability model based on this technique is

$$R(d) = \exp \left[-(b_1d + b_2d^2 + \dots + b_jd^j) \right], \quad d \geq 0$$

where the degree of the polynomial j and the coefficients b_i ($i = 1, \dots, j$) are determined from a statistical multiple-regression analysis.

(3) Numerical Example

The multiple-regression technique of failure-rate estimation is applied herein to a set of 2N718A transistors tested in a radiation environment. For convenience in handling, the raw data have been coded so that 1 unit equals 10^4 rad(C). A linear plot of the failure data in Table IX (see Fig. 18) and the fact that the graphical estimate of the Weibull shape parameter for

Table IX
VARIABLES AND NORMAL EQUATIONS
OF THE REGRESSION ANALYSIS

Rank Number 1	$R(\hat{d}_1)$	Regression Variables			
		$-\ln R(\hat{d}_1)$ $y(d_1)$	d_1	d_1^2	d_1^3
--	1.00	0.0000	0.00	0.00	0.00
1	0.91	0.0943	1.39	1.9321	2.6856
2	0.82	0.1985	2.48	6.1504	15.2530
3	0.73	0.3147	2.58	6.6564	17.1735
4	0.64	0.4463	3.50	12.2500	42.8750
5	0.55	0.5978	3.55	12.6025	44.7389
6	0.45	0.7985	3.60	12.9600	46.6560
7	0.36	1.0217	4.30	18.4900	79.5070
8	0.27	1.3093	4.40	19.3600	85.1840
9	0.18	1.7148	5.30	28.0900	148.8770
10	0.09	2.4080	5.76	33.1776	191.1030
 <u>Normal Equations to be Solved</u>					
$151.6690b_1 + 674.0530b_2 + 3,169.2063b_3 = 41.1069$					
$674.0530b_1 + 3,169.2061b_2 + 15,548.5576b_3 = 199.1470$					
$3,169.2063b_1 + 15,548.5576b_2 + 78,813.8073b_3 = 1,000.0543$					

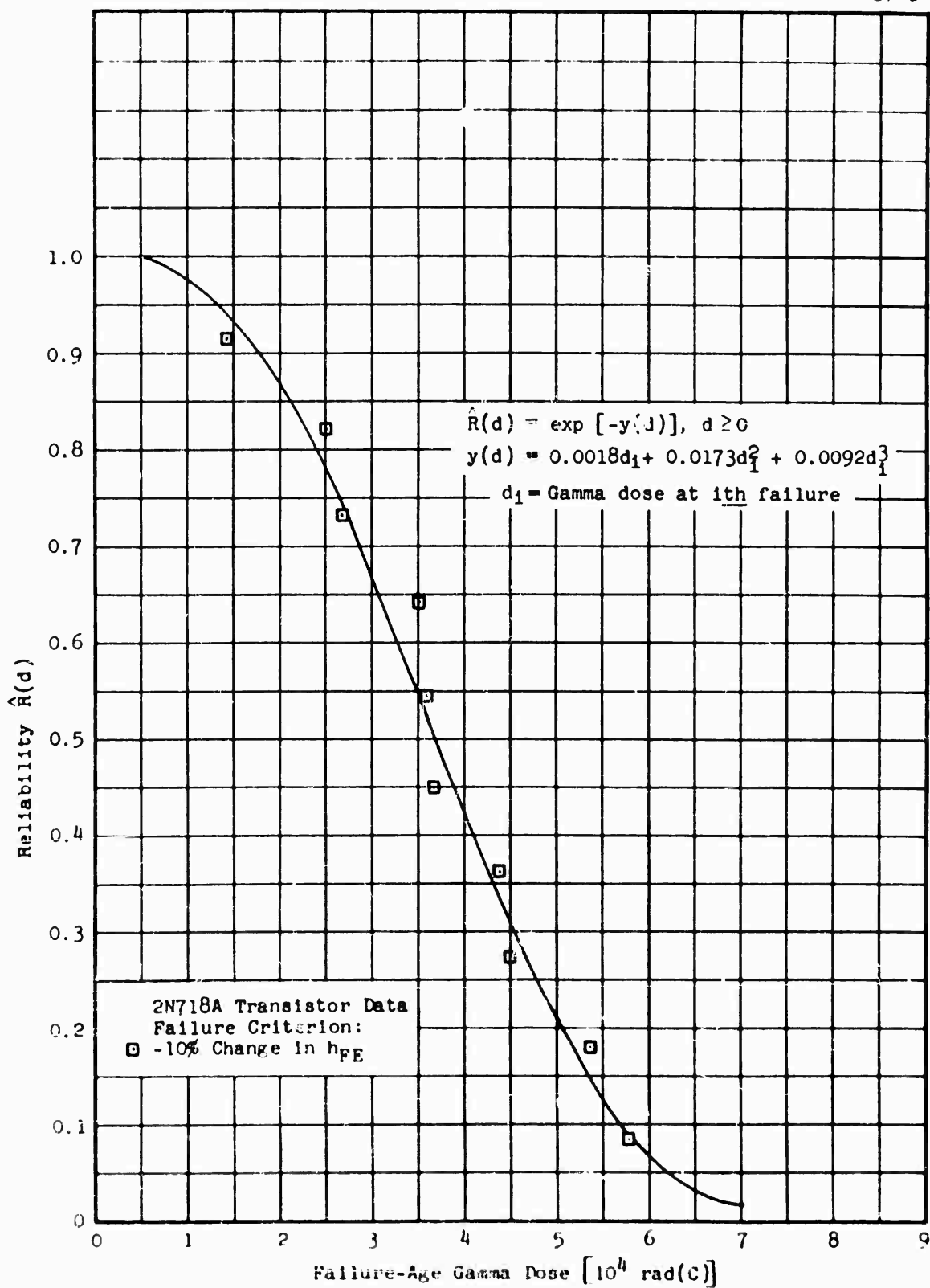


Figure 18 A Reliability Model Based on Failure-Rate Regression Analysis

this set of data is $\hat{\beta} = 2.5$ suggest that the approximating polynomial should be of degree 3 or less. Consequently, the coefficients for a cubic equation that is forced through the origin will be determined and the appropriate test of significance will be performed.

The cubic multiple-regression equation based on the data of Table IX is

$$y(d) = 0.0018d + 0.0173d^2 + 0.0092d^3 \quad (11)$$

The error mean square associated with this equation is 0.0029. The pivotal method (Ref. 11) was employed to solve the normal equations of the regression analysis (see Table IX). One of the purposes of the analysis is to determine the appropriate degree of the polynomial; this requires an analysis-of-variance for which the pivotal method provides the required information. Another advantage of the pivotal method is that it successively fits the dependent variable and, if an analysis-of-variance finds the j th degree to be insignificant, then no additional computation is necessary to obtain the coefficients for the polynomial of degree $j-1$.

If the assumptions basic to the standard multiple-regression analysis are valid - that is,

1. The variance of the error is approximately constant (independent of the controllable variables)
2. The errors for the different observations are statistically independent, and
3. The error distribution is approximately normal - then approximate tests of significance can be made by use of

the data obtained from the pivotal method of solving the normal equations. The information required is presented in Table X.

Table X
ANALYSIS OF VARIANCE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio
d (linear)	1.7112	1	1.7112	$\frac{1.7112}{0.0029} = 590$
d ² after d (quadratic)	9.5746	1	9.5746	$\frac{9.5746}{0.0029} = 3,302$
d ³ after d and d ² (cubic)	1.5322	1	1.5322	$\frac{1.5322}{0.0029} = 528$
Error	0.0206	7	0.0029	
Total	12.8386	10	$F_{99\%}(1,7) = 12.25$	

Of primary interest is the determination of the appropriate degree of the approximating polynomial. The tabular value of an F ratio for one and seven degrees of freedom at the 99% level of significance is 12.25. Comparing this to the F ratio in Table X for the cubic effect gives evidence at the 99% level that $b_3 \neq 0$ and, hence, the cubic term is significant.

A check of Equation 11 reveals that conditions 9b and 9c are also satisfied. Consequently, the reliability model based upon the failure-rate regression technique is

$$R(\hat{d}) = \exp \left[- (0.0018d + 0.0173d^2 + 0.0092d^3) \right], \quad d \geq 0$$

Figure 18 illustrates graphically the goodness-of-fit of this reliability model.

(4) Further Considerations

The analysis-of-variance indicated that the cubic term of the polynomial, $y(d)$, was statistically significant at the 99% confidence level. However, for illustrative purposes, results of the linear and quadratic equations, also acquired from the regression analysis, are presented and compared to the cubic equation in Figure 19. This comparison graphically illustrates the increasing improvement in the goodness-of-fit of the reliability model with the inclusion of higher-order terms in the approximating polynomial.

In the regression analysis technique of reliability estimation, which has been presented herein, the only restriction imposed upon the polynomial $y(d)$ before the regression analysis is performed is $y(0) = 0$, which implies $b_0 = 0$. With this single restriction, the standard techniques of multiple regression may be employed to determine $y(d)$. However, this single restriction on $y(d)$ is not sufficient to ensure that all necessary conditions upon $y(d)$ are met. In the case of the cubic equation, all of the coefficients obtained from the regression analysis were positive, and the solution of $y(d)$ was such that the additional conditions on $y(d)$ - that is, $9b$ and $9c$ - were also satisfied.

However, in the case of the quadratic equation, the regression analysis gave one negative coefficient and, as a result, condition $9c$, below, is violated:

$$y'(d) \geq 0 \quad \text{all } d \geq 0.$$

Figure 19 illustrates the effect of this violation upon the reliability model in the interval $0 \leq d \leq 1.25$. That is, it produces a reliability function with a positive slope and a reliability estimate greater than 1, and both are, of course, impossible. This example illustrates the necessity of verifying that any equation obtained from this procedure also satisfies conditions 9b and 9c. If the equation obtained from the standard regression techniques does not satisfy the additional conditions 9b and 9c, it must be rejected or adjusted so that it will not effect such a violation.

There is a modification of the standard multiple-regression analysis which assures that all necessary conditions on $y(d)$ are met with the first determination of the polynomial. This is accomplished by placing the following conditions upon the coefficients of the polynomial,

$$b_0 = 0$$

$$b_j > 0$$

$$\sum_{i=1}^j 1b_i^{i-1} \geq 0, \quad d \geq 0$$

and modifying the regression procedures so that these conditions will be met, and, consequently, the conditions 9a, 9b, and 9c will be satisfied. The techniques of this modified regression procedure are presented in an article written by Krane (Ref. 12).

3. The Exponential Assumption

The estimates of the Weibull shape parameter, β , obtained

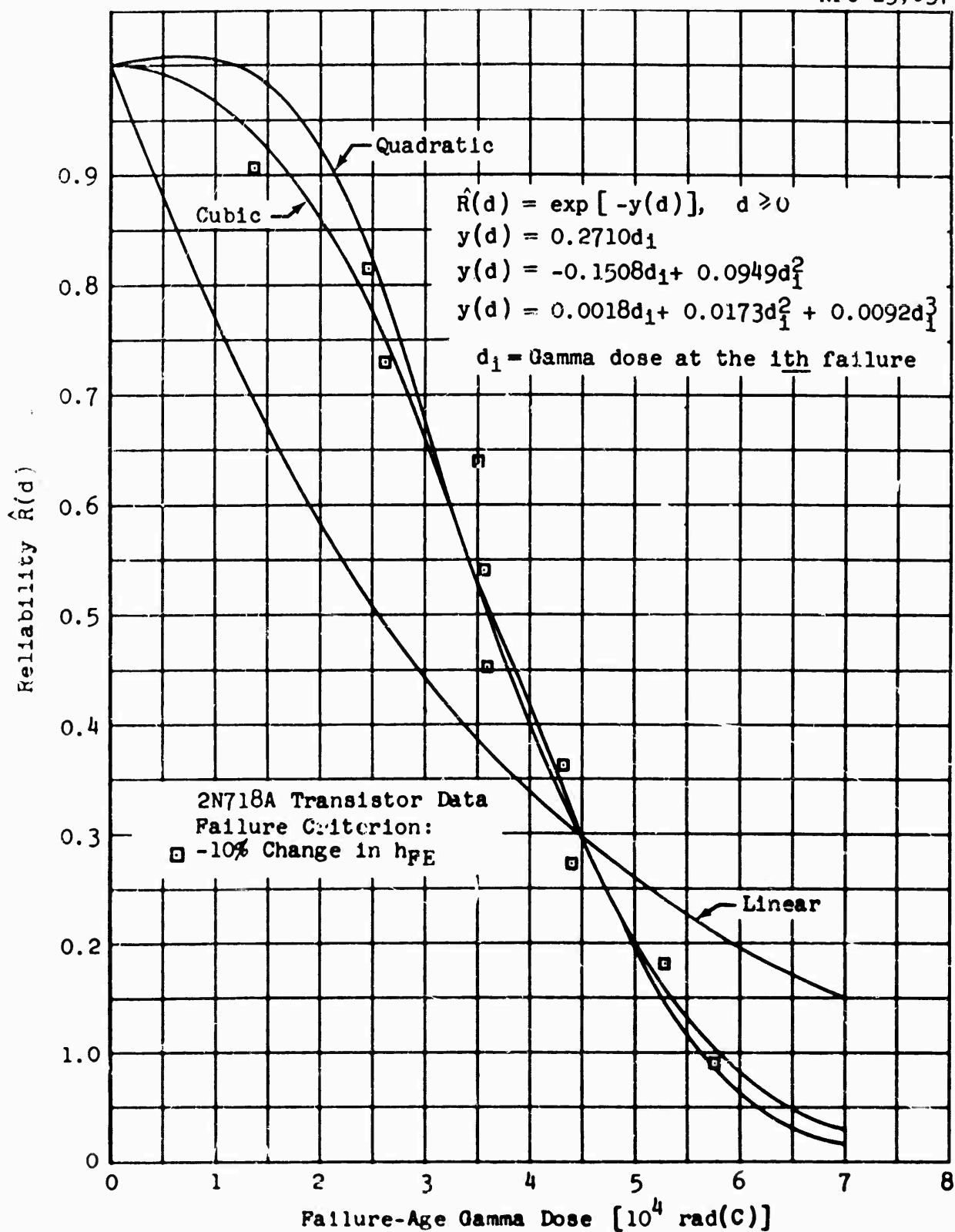


Figure 19 Effect of Higher-Order Terms in the Reliability Regression Model

in this study for transistors, diodes, and capacitors operating in a radiation environment are predominantly greater than unity ($\beta = 1$ being the exponential distribution). Others have made this observation on the same and other classes of electronic devices operating in various environments; for example, Kao (Ref. 13) has found that for a certain class of electron tubes a value of 1.7 may be quite appropriate. ARINC (Ref. 14) concludes that estimates of β for the transistor types studied in 1,000-hr life-tests are predominantly less than unity. Even with evidence that β is not unity, the exponential assumption is still used. This study hypothesizes that any "good" estimate of β will certainly be better than the total acceptance of the exponential distribution. The only legitimate question that can be raised in opposition to this philosophy is the desirability of past data from which β has to be estimated. However, these data are becoming more numerous and available from past records of companies that are engaged in similar types of research.

When the exponential distribution is assumed, in lieu of radiation reliability test data in sufficient quantities to statistically determine the true value of β , it is supposedly justified upon the following bases:

1. There exist experimental data which support the exponential distribution as the lifetime distribution for similar components operating in a nonradiation environment.
2. The exponential distribution is a conservative assumption when the alternative is a Weibull distribution with $\beta > 1$.

The nomenclature "conservative assumption" is derived from the

fact that if the underlying failure distribution is a Weibull distribution with $\beta > 1$, rather than the assumed exponential, stronger probability statements could be made. That is, for any given set of test data, the reliability estimates from the data will be less if the underlying failure distribution is exponential than it would be if the underlying distribution were Weibull with $\beta > 1$; thus, the exponential assumption, if in error, results in conservative probability estimates. While this philosophy can be viewed as being conservative in its reliability estimates, it can also be said that it is extremely erroneous in the statement of true test errors if the assumption is false. The comparison of reliability estimates for $\beta = 1$ and $\beta > 1$, along with the problem of rejecting reliable equipment because of an erroneous exponential assumption, is now considered.

a. Comparison of Reliability Estimates ($\beta = 1$ vs $\beta > 1$)

The effect of erroneously accepting the exponential ($\beta = 1$) as the underlying failure distribution when it is in fact a Weibull distribution ($\beta > 1$) can be illustrated by comparing the reliability estimates obtained from actual test data under each of the failure distributions. By selecting the method of moments as the means of parameter estimation, it can be shown that for any given set of test data, the distribution $\beta > 1$ yields higher reliability estimates for all reliabilities greater than 0.368 than the distribution $\beta = 1$ (Ref. 15).

The Weibull distribution ($\gamma = 0$) is defined as

$$f(t) = \frac{\beta t^{\beta-1}}{a} \exp \left[-t^{\beta} / a \right]$$

Using the method of moments for parameter estimation requires that the two equations

$$\bar{t} = \hat{\alpha}^{1/\hat{\beta}} \Gamma(1 + 1/\hat{\beta})$$

and

$$s^2 = \hat{\alpha}^{2/\hat{\beta}} \left[\Gamma(1 + 2/\hat{\beta}) - \Gamma^2(1 + 1/\hat{\beta}) \right]$$

be solved simultaneously for the estimates of α and β [see Section II-2c(1)]. However, if the value of β is known, the estimate of α can be expressed as a function of the sample mean only:

$$\hat{\alpha} = \left[\bar{t} / \Gamma(1 + 1/\beta) \right]^\beta$$

In this situation, the α estimates for $\beta = 1$ and $\beta > 1$ are respectively,

$$\hat{\alpha}_1 = \bar{t}$$

$$\hat{\alpha}_2 = \left[\bar{t}/p \right]^\beta$$

where p is a fraction greater than zero but less than 1:

$$p = \Gamma(1 + 1/\beta)$$

To determine which distribution results in the highest reliability estimate for any set of experimental data, compare the exponents of the following reliability models, which represent the $\beta = 1$ and $\beta > 1$ cases, respectively:

$$R(t) = \exp(-t/\hat{\alpha}_1), \quad \beta = 1$$

$$R(t) = \exp(-t^\beta/\hat{\alpha}_2), \quad \beta > 1$$

<u>For $\beta = 1$</u> $(t / \hat{\alpha}_1)$ $(\frac{t}{\bar{t}})$ $(\text{For } t \leq \bar{t})$ $(\frac{t}{\bar{t}})$	<u>For $\beta > 1$</u> $(t^\beta / \hat{\alpha}_2)$ substituting for $\hat{\alpha}$ $\left[\frac{t^\beta}{(\bar{t}/p)^\beta} \right]$ $(p \cdot \frac{t}{\bar{t}})^\beta$
---	---

Consequently, for any set of test data, the reliability estimates for $\beta = 1$ are less than the similar estimates for $\beta > 1$, i.e.,

$$\exp \left[-(pt/\bar{t})^\beta \right] \geq \exp (-t/\bar{t}), \quad \text{for } t \leq \bar{t}$$

Thus, if for a given test result the method of moments is used for estimating the lifetime parameter α , the Weibull reliability function, $\beta > 1$, will result in a higher reliability estimate than the exponential reliability function, $\beta = 1$, for all reliabilities greater than $\exp(-1) = 0.368$. It is on this basis that the exponential assumption is termed conservative.

b. The Robustness of Life-Testing Procedures

In the area of experimental design and subsequent testing programs, it is often necessary to make various assumptions regarding the nature of the phenomena under study. When data analysis is based upon such assumptions, one of the following courses of action should be pursued:

1. The assumptions should be verified either analytically or experimentally.
2. The sensitivity of the analysis to the underlying assumptions should be revealed. (Some statistical

procedures are very insensitive to departures from the basic assumptions. This highly desirable quality is termed "robustness.")

In this study, it was not possible to follow the first course of action. Consequently, the sensitivity of the reliability analysis to the commonly assumed exponential distribution must be determined.

To illustrate the sensitivity of a radiation reliability testing program to an erroneous exponential assumption, a comparison of experimental testing errors is presented. The comparison is between the $\beta = 1$ assumption and the case when β is in fact greater than 1.

(1) Experimental Testing Errors

In life-testing programs, the decision of whether or not to accept the equipment in question as being reliable is based upon a set of statistics obtained from the test data. The possible outcomes of a life-testing experiment and the consequences of a decision made are presented in Table XI.

Table XI
POSSIBLE OUTCOMES OF A TESTING PROGRAM

Decision: Based Upon Test Statis- tics	Unknown to the Experimenter the Equipment is:	
	Reliable	Unreliable
Accept Equipment as Being Reliable	Correct Decision	(Type II Error)
Reject Equipment as Being Unreliable	(Type I Error)	Correct Decision

Examination of Table XI reveals that two different types of errors can be committed:

Type I error - the rejection of reliable equipment

Type II error - the acceptance of unreliable equipment

"Reliable equipment" is defined to be equipment that has a specified probability p of exceeding a particular mission duration t_0 . That is, if $R(t_0) \geq p$, the equipment is reliable; if not, the equipment is unreliable.

Obviously, for a testing program to be of any significance the magnitude of the Type I and Type II errors must be controlled by the experimenter. Let η and ξ represent the probabilities of committing a Type I error and a Type II error, respectively. The ideal testing program would be one in which the size of both η and ξ are an absolute minimum. Unfortunately, these two errors work in opposite directions: for any fixed sample size, changing the decision criteria to decrease the probability of a Type I error increases the probability of a Type II error, and vice versa.

However, by consideration of the resources available and the relative seriousness of the Type I and Type II errors, it is possible to determine the most acceptable ratio between the sizes of the two errors. On this basis, it is possible to establish the maximum η and ξ that can be tolerated and to design the test program so that these maximum values will not be exceeded. The most commonly used procedure for determining a reliability life-testing program that will meet the particular η and ξ

specification is as follows:

1. Assume that the underlying failure distribution is some form of the exponential distribution.
2. Define reliable equipment and unreliable equipment.
3. Select the desired type of the life-test: (1) fixed sample size, (2) truncated replacement and nonreplacement, or (3) sequential.
4. Using the relationships and/or tables given in Reference 16, look up the test plan specifications that will give the desired η and ξ .

(2) Reliability Expressed in Terms of Mean Life

Associated with each test plan is an operating characteristic (O.C.) curve, which expresses the probability of accepting the equipment as a function of some population parameter. When dealing with reliability estimation, the population parameter most commonly employed in the O.C. curves is the mean lifetime.

One recommended procedure (see Ref. 17) for exponential life testing is to design the test so that (1) reliable equipment will be rejected 10% of the time and (2) unreliable equipment will be accepted 10% of the time (i.e., $\eta = \xi = 0.1$), where "reliable equipment" is defined as equipment that has a mean life $\alpha \geq m$ and "unreliable equipment" is defined as equipment which has a mean life $\alpha \leq m/2$. Suppose that the equipment design goal is a mean lifetime of 1,000 hr. For the exponential distribution, this is equivalent to the design goal of a reliability for 105 hr ≥ 0.90 , or $R(105) \geq 0.90$. Using the life-testing plan described above, "reliable equipment" is defined to be that equipment for which the mean life is equal to or greater than 1,000 hr, and "unreliable equipment" is that for which the mean life is equal to or less

than 500 hr. Thus

$$0.1 = \eta = \Pr(\text{reject equipment}/\hat{a} = 1,000 \text{ hr}) \quad (12)$$

$$0.1 = \xi = \Pr(\text{accept equipment}/\hat{a} = 500 \text{ hr}) \quad (13)$$

If a fixed-sample-size test procedure is to be used for the life-testing program, the only parameter to be determined is the sample size n . The conditions expressed in Equations 12 and 13 will be satisfied if the sample size n is taken to be the smallest interger, so that

$$\chi^2_{1-\eta}(2n)/\chi^2_{\xi}(2n) \geq 1/2$$

where $\chi^2_{1-\eta}(2n)$ denotes the lower η percentile of the chi-square distribution for $2n$ degrees of freedom, and n equals the number of failures.

The rule for accepting or rejecting the equipment as being reliable is based on the test statistic \hat{a} . Given

$$\hat{a} = \frac{1}{n} \sum_{i=1}^n t_i \quad \text{and } C = 1,000 \chi^2_{1-\eta}(2n)/2n$$

accept the equipment if $\hat{a} \geq C$ and reject the equipment if $\hat{a} < C$. To meet the specifications of Equations 12 and 13, n must be equal to 14 and, consequently, $C = 676$. A complete discussion of the life-testing procedures is given in Reference 16.

The O.C. curve for this testing plan is presented in Figure 20. If the underlying failure distribution is exponential, as assumed, the correct test error probabilities are taken from the O.C. curve for $\beta = 1$; if, however, the failure distribution is Weibull $\beta > 1$, the error probabilities are given by the appropriate O.C. curve for which $\beta > 1$. The O.C. curves presented in Figure 20 were taken from Reference 17.

Reliable Equipment Based on the
Exponential Distribution is
 $R(105) \geq 0.90$

Actual Reliability Distribution is
 $R(t) = \exp(-t^{\beta}/\alpha)$

Fixed-Sample-Size Test Plan
Based on the Exponential
Distribution
($N = 14$)

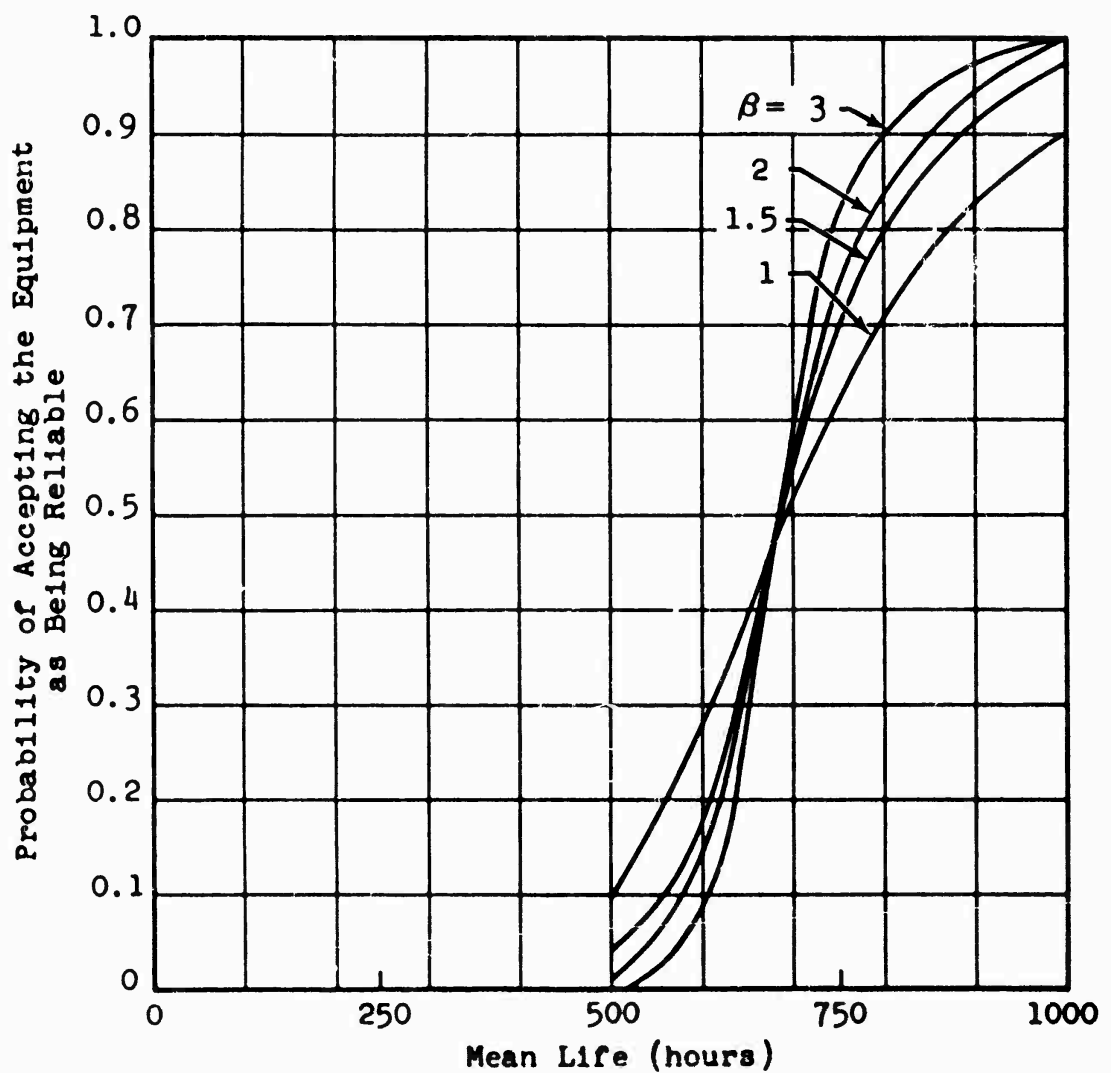


Figure 20 Operating-Characteristic Curves Expressed as a Function of Mean Life

(3) Reliability Expressed in Terms of a Percentile

The O.C. curves of Figure 20 are presented as a function of the equipment mean lifetime. However, expressing reliability in terms of mean lifetime can be misleading when distributions other than the exponential are compared. To illustrate this, express the Weibull reliability function

$$R(t) = \exp(-t^\beta / \alpha) \quad \gamma = 0$$

as a function of the mean lifetime μ . Since

$$\mu = \alpha^{1/\beta} \Gamma(1 + 1/\beta)$$

then
$$R(t) = \exp\left[-t \Gamma(1 + 1/\beta) / \mu\right]^\beta$$

The minimum mean lifetime for which the reliability $R(t)$ is equal to or greater than p is

$$\mu = t \Gamma(1 + 1/\beta) [-\ln R(t)]^{-1/\beta}$$

Consequently, the minimum mean life values for which $R(105) \geq 0.90$ (the previously stated design goal) are 1,000 and 287 hr for $\beta = 1$ and $\beta = 2$, respectively.

Obviously, when comparing the exponential distribution with Weibull distributions ($\beta > 1$), mean lifetime specifications are not sufficient. Therefore, it is suggested that reliability specifications be given in terms of percentiles or a particular point on the reliability curve, rather than mean lifetimes. This leads to the construction of O.C. curves as a function of actual reliability rather than the misleading mean lifetime. Such an

O.C. curve will then express the probability of accepting the equipment under the test plan as a function of the equipment's actual reliability for some given time, t_0 , rather than as a function of mean lifetime.

As an illustrative example, the O.C. curves of Figure 20 are redrawn as a function of equipment reliability rather than the mean lifetime, where reliability is defined to be the probability of the equipment's life exceeding 105 hr (see Fig. 21).

The effect of the "conservative" exponential assumption upon life-testing errors is illustrated by the O.C. curves of Figure 21. It is seen that for the $\beta = 1$ curve, the probability of rejecting equipment with a 90% or greater reliability is equal to or less than 0.1, and the probability of accepting equipment with a reliability of 80% or less is equal to or less than 0.1; that is, $\eta = \xi = .1$. These are the specifications for which the test procedure was designed, based upon the exponential assumption. However, if the exponential assumption is in error and the failure distribution is actually Weibull with, for example, $\beta = 2$, the probability of rejecting equipment with 90% reliability is approximately 1 and the probability of accepting equipment with 80% reliability is approximately zero; that is, if $\beta = 1$ is assumed, but unknown to the investigator, $\beta = 2$, then the testing errors are $\eta \approx 1$ and $\xi \approx 0$. Further, if the exponential assumption is false (i.e., $\beta = 2$) and the actual reliability is in the range 0.90 to 0.95, the probability of accepting it as reliable equipment is virtually zero, when in fact it is more reliable than the original specification called for (i.e.,

Reliability Equipment $R(105) \geq 0.90$
 Actual Reliability Distribution is
 $R(t) = \exp[-t^\beta/a]$
 Fixed-Sample-Size Test Plan Based
 on the Exponential Distribution
 (N = 14)

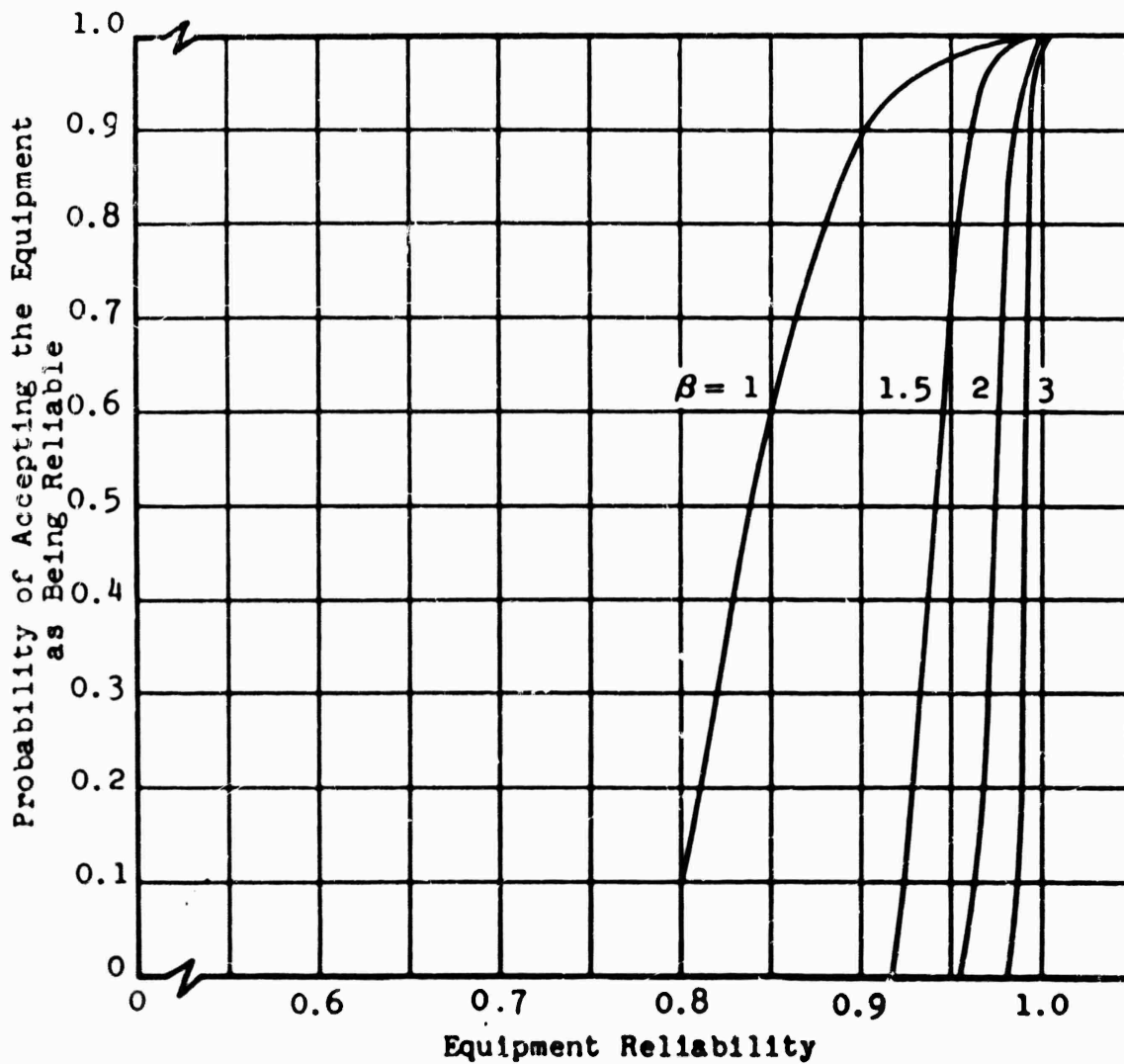


Figure 1.1 Operating-Characteristic Curves Expressed as a Function of Reliability

$R(105) \geq 0.90$). Even with a reliability as high as 0.98, there is only a 0.75 probability of accepting that equipment as being reliable under this test plan, which is based on a false exponential assumption.

The problem above considered only a single case ($\beta = 1$ vs $\beta = 2$); however, it should be recognized that the same general relationships hold when the exponential distribution is compared with any Weibull distribution ($\beta > 1$). The contrast between the O.C. curves, presented as a function of mean life and as a function of reliability $[R(t_0) \geq p]$, will be a minimum for $\beta = 1$ vs $\beta = 1 + \epsilon$ and will steadily increase as ϵ increases. The problem above is the special case for which $\epsilon = 1$.

In conclusion, the effects of an erroneous exponential assumption upon (1) life-test reliability estimates and (2) experimental testing errors, when in fact the true failure distribution is Weibull ($\beta > 1$), are (a) that the reliability estimates based upon life-test data are understated and (b) that the subsequently designed life-testing programs have a high probability of rejecting reliable equipment.

c. Discussion

The following discussion is a brief review of the limitations of the statistical failure data analysis performed herein.

When dealing with small sample sizes for determining the "best" failure distribution to represent the data, there are no statistical tests which are powerful enough to discriminate between all possible a priori distributions.

There is no analytical solution to the problem of placing

confidence limits upon the Weibull shape parameter, β , when the population parameters are unknown. To design a test program with specified testing errors (Type I and Type II) requires a knowledge of the underlying failure distribution. For the Weibull distribution, β must be known within limits. Preliminary studies employing Monte Carlo techniques have been conducted to determine the distribution of α and β for the Weibull distribution (Ref. 14).

SECTION III

DATA ANALYSIS

The preceding section set forth the basic concepts of statistical analysis and discussed analytical models that can be used to describe radiation reliability relationships. This section was performed concurrently with the mathematical models section, but the significance of the results presented here can best be realized with an understanding of the basic statistical techniques and mathematical considerations described in Section II.

The initial approach to the problem of obtaining data for this study was to determine which subsystems, circuits, and components would be used in the proposed systems utilizing nuclear auxiliary power and then to collect data on these items. However, it was soon discovered that a more practical approach was to first collect the very limited nuclear component data that had sufficient sample sizes. After the usable nuclear data were found, the corresponding life-test nonradiation reliability data were obtained from the manufacturer. The bulk of the radiation data was from the NARF-Lockheed Missiles and Space Company (LMSC) tests.

The data were treated by various methods of analysis to determine their sensitivity to various environments and to evaluate their characteristics relative to various statistical parameters, such as failure-distribution functions, failure criteria, sample sizes, mean time to failure, and test duration. The most qualified data found in the data analysis were used in the preceding mathematical models study.

1. General Procedures

a. Treatment of Raw Data

Analysis and interpretation in the sensitivity analysis rely heavily upon rank methods (Refs. 18, 19, and 20) because the data available are not sufficient to justify an assumption regarding the underlying failure distribution. Although rank methods do not fully utilize the information contained in the data, they are advantageous because they are easy to apply and application of the methods does not require any assumption concerning the form of the sample distribution(s).

It must be noted that statistical (rank) methods determine only the statistical significance of the difference between the sets of data. An effect may be statistically significant and be of no engineering importance. When an effect is termed not significant, it does not necessarily mean that there is no effect. It may be that the experiment was not sensitive enough to detect an existing effect.

The significance tests and probability statements based on rank methods for evaluating the effects of different environments are used in this study with the knowledge, of course, that many of the experiments may not have been conducted in the manner implied by the significance tests. That is, randomization procedures may have been violated and the conditions analyzed may not have been the actual objectives of the experiment. Therefore, any seemingly significant outcome that was not taken into account in advance of performing the experiment can only suggest a new experiment. It

follows that an experimenter who does not anticipate any conclusions at all, but merely waits to see what will turn up in the data, cannot support any conclusion whatever by a probability statement.

b. Failure Criteria

Only catastrophic and degradation failures are considered in this report. Degradation failures occur when the gradual change in a measured characteristic exceeds some specified upper or lower limit(s). One failure criterion is sufficient for catastrophic failures, but not for degradation failures. A realistic degradation failure criterion is actually dictated by the use of the device; therefore, more than one failure criterion was considered for component degradation failures.

In a reliability investigation, it is the observed failures that constitute the data for an analysis, not the number of units tested. In most instances, 1,000-hr life-tests produced too few failures for an analysis, even though from 100 to 300 units were used. Most of the reliability plots of life-test data are the result of using a more severe failure criterion than that used in the life-test specifications. This was done to create some failures by definition and thus gain an insight into the reliability of life-test data. In some cases, the estimated failure rate quoted by the manufacturer, along with the exponential assumption, is considered for comparison purposes. This approach is used for the radiation data analyzed on a reliability basis. When there are no radiation degradation and/or catastrophic failures, the data are not considered in the analysis.

c. Presentation of Data

The radiation reliability data are presented predominantly as Weibull plots. This form of presentation was chosen on the basis of its use in other related fields of failure analysis and the known versatility of the model and its corresponding graph paper for presenting reliability data. A detailed explanation of the Weibull failure distribution and its use is presented in Sections II-1c and II-2.

2. Sensitivity of Temperature and Correlation Coefficient

Table XII contains radiation-temperature data from the NARF-LMSC tests on gain, h_{FE} , for 2N1613 transistors. Column 1 contains the initial h_{FE} values for radiation test temperatures of 86° and 140°F. Column 2 contains the rank serial numbers 1, 2, 3,... assigned to the actual gain values in column 1. The lowest gain value is assigned a rank of 1, the next highest a rank of 2, etc. Since the h_{FE} values corresponding to the 4th and 5th rank numbers are the same (38), both are assigned the median rank value of 4.5. Column 3 is the recorded gamma dose at which the corresponding transistor gain changed 40% from its initial value (failure criterion: -40% change in h_{FE}). Column 4 contains the ranks of the gamma doses in column 3. Column 5 contains the gamma dose at which the transistor gain decreased to 20 (failure criterion: minimum h_{FE} = 20). Column 6 contains the ranks of the gamma doses in column 5.

A significance test based on rank methods was performed on the ranked temperature data in columns 2, 4, and 6. The sum of ranks for the 86°F data was compared with the sum of ranks for the 140°F

Table XII
APPLICATION OF RANK METHOD TO RADIATION TEMPERATURE
DATA FROM 2N1613 TRANSISTORS

Initial Gain, h _{FE} , at Indi- cated Tempera- ture	Corresponding Rank No.	Gamma Dose Effecting 40% Change in h _{FE} [10 ⁵ rad(C)]	Corresponding Rank No.	Gamma Dose at Which h _{FE} = 20 [10 ⁵ rad(C)]	Corresponding Rank No.
86°F					
27	1	3.6	10	.75	1
38	4.5*	0.62	5	1.5	2
28	2	3.4	8.5	1.8	3
61	9	0.55	2	7.1	8
50	6	2.9	7	11	9
Sum of Ranks	22.5		32.5		23
140°F					
54	7	0.60	3.5	5.9	7
38	4.5	3.4	8.5	4.3	6
60	8	0.38	1	3.8	5
68	10	0.60	3.5	14.7	10
36	3	1.8	6	2.6	4
Sum of Ranks	32.5		22.5		32

*Tied Values have been assigned median rank values.

data within each column to determine, on a probability basis, whether the observed differences in the sums were due to a significant temperature effect or to chance (not significant or no detectable temperature effect). From the results of these tests it was concluded that there were no significant temperature effects that could be statistically determined from the observed data. Thus, the radiation temperature data were combined and plotted as Weibull plots in Figure 22.

Estimates of the rank correlation coefficient, r' , were also determined from the ranked data in Table XII to show the association between (a) the initial h_{FE} and the radiation-induced failure at the i th gamma dose effecting the 40%-change failure criterion, r'_1 ; and (b) the initial h_{FE} and the radiation-induced failure at the i th gamma dose effecting the minimum $h_{FE} = 20$ failure criterion, r'_2 . This rank correlation method is used to estimate the correlation between the positions or ranks of each pair of observations. The ranks are assigned from lowest to highest when the observations on each variable are arranged in order of magnitude. This method is useful as a rapid test of the association between two variables and is generally satisfactory when applied to data of this type because it is nonparametric - i.e., the normality of the distribution sampled need not be assumed.

An estimate of the correlation between the ranks in columns 2 and 4 gives $r'_1 = -0.80$. This value of r'_1 is significant at a confidence probability ≥ 0.99 . The interpretation is that there is a significant negative correlation, and there is less than one

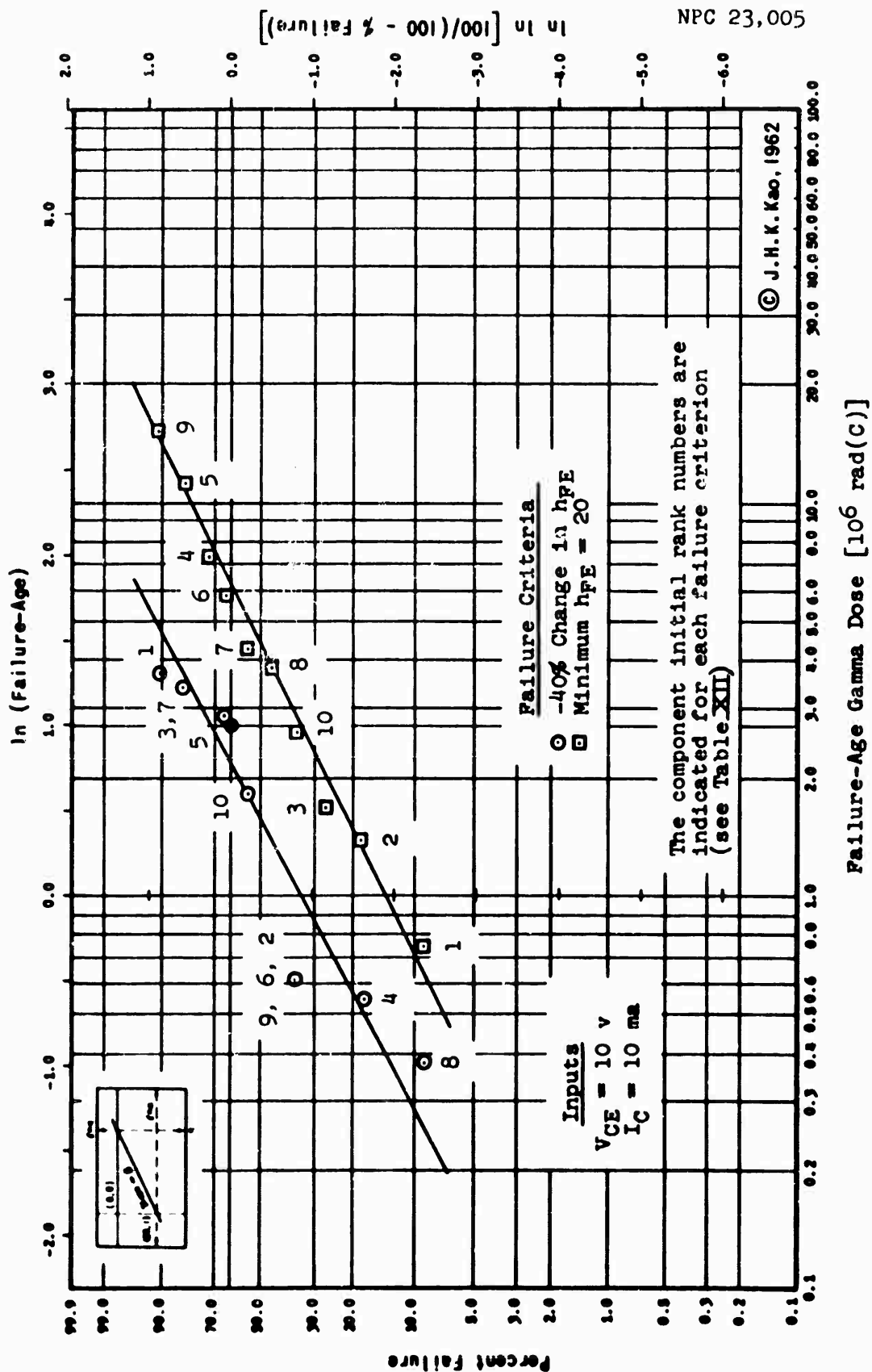


Figure 22 Weibull Radiation-Reliability Plot of h_{FE} for TI 2N1613 Transistors

chance out of 100 that an observed negative correlation as large or larger could have occurred by chance as a result of experimental error.

An estimate of the correlation coefficient between the ranks in columns 2 and 6 gives $r_2' = +0.85$, with a confidence probability ≥ 0.99 . (This is a similar interpretation as above except for positive r' .)

The difference in the signs (positive and negative) for the estimates of r_1' and r_2' can be understood by noting in Table XII the order of the initial rank numbers and comparing this order with those of r_1' and r_2' in Figure 22. The order of failures for r_2' tends to follow the order of the initial rank numbers and shows up as a positive correlation, whereas the order of failures for r_1' tends to be the reverse of the initial rank numbers and shows up as a negative correlation. These estimated correlation coefficients are for off-the-shelf transistors and would probably not be as large or, in some cases, not even exist had the transistors been selected with initial h_{FE} values clustered closely about a typical value. That is, if 10 transistors with initial h_{FE} values between 60 and 65 had been selected, r_1' and r_2' might well be zero. This sensitivity (as measured by the positive correlation coefficient r_2') of the h_{FE} values at the i th nuclear exposure to their initial values, as measured on an absolute scale, was prevalent in the data analyzed. The sensitivity was more pronounced when the initial h_{FE} values covered a wide range.

When the h_{FE} values at the i th nuclear exposure were normalized by their initial values, the sensitivity to the actual initial gain

values was not as pronounced nor as consistent. In some cases, the estimate of correlation coefficient r_1 was positive and in other cases it was negative. Again it was noted that when the sensitivity on a normalized scale occurred, there was a wide range of initial h_{FE} values.

When the initial h_{FE} values cover a wide range of values it presents a problem in the analysis of the data in choosing the scale of measurement (absolute or normalized) upon which to base conclusions. If there had been a significant temperature effect, and the correlation values had remained the same, then in one scale of measurement there would be for the same data a positive temperature effect and in the other a negative temperature effect. Thus, when a wide spread in h_{FE} values occurs, the conclusions that can be drawn are quite sensitive to the scale of measurement (normalized or absolute) and, in this case, the conclusions are sensitive to the failure criteria for analyzing h_{FE} . The criteria used will depend upon whether the designer is using a minimum gain or a percent change.

It follows that if one is designing on a minimum-gain basis, then a transistor from this group should be chosen with the highest initial gain that can be tolerated in the circuit.

Figure 23 shows the individual Weibull plots of the -40% h_{FE} temperature data for the TI 2N744 transistors. The combined-temperature plots of these reactor test data (gamma dose scale), along with some 2N744 Co⁶⁰ Atomics International test data, are presented in Figure 24. Figure 25 contains Weibull plots on a neutron scale for

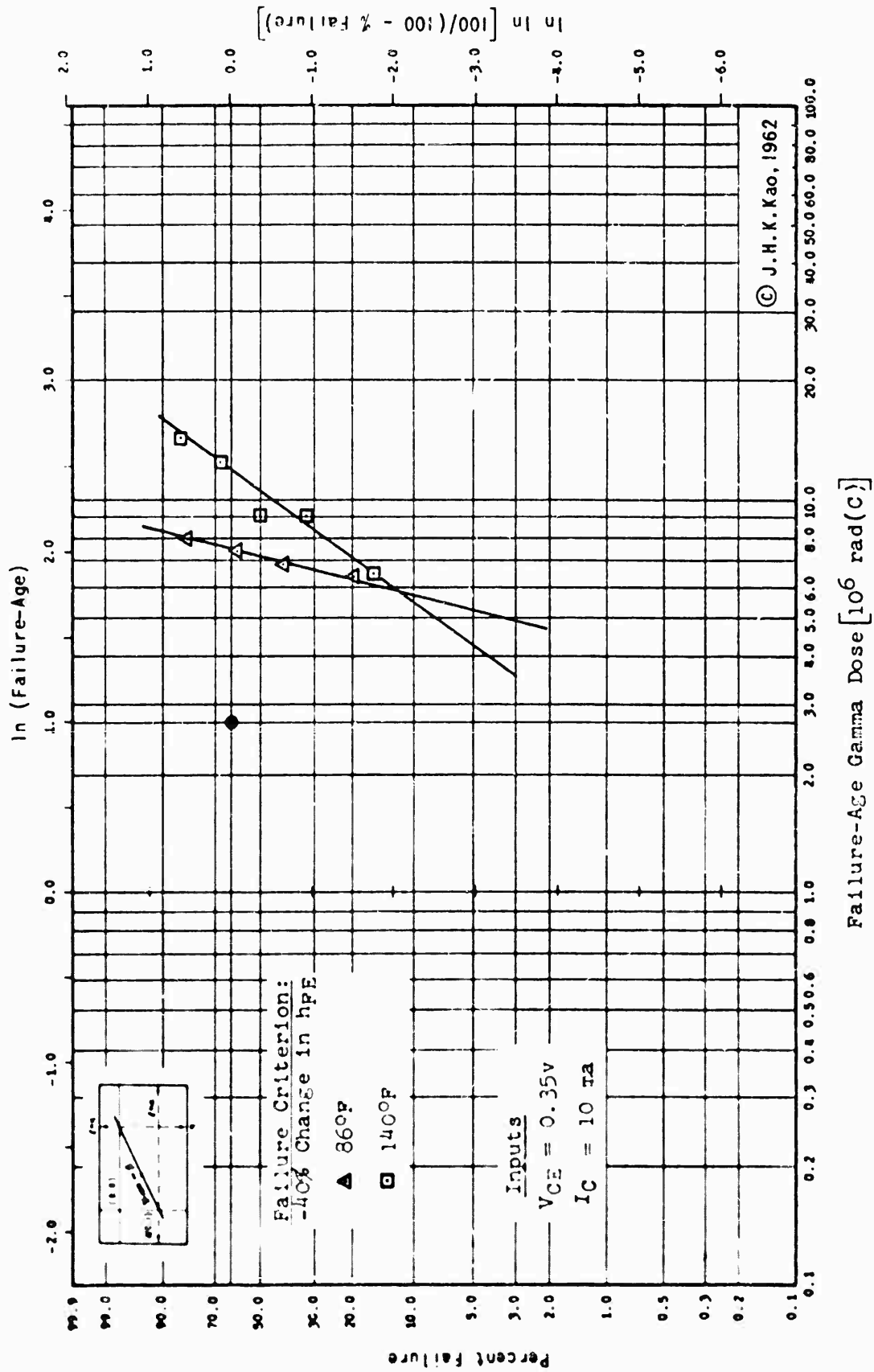


Figure 23 Weibull Radiation-Reliability Plot of h_{FE} for TI 2N744 Transistors for Two Temperatures

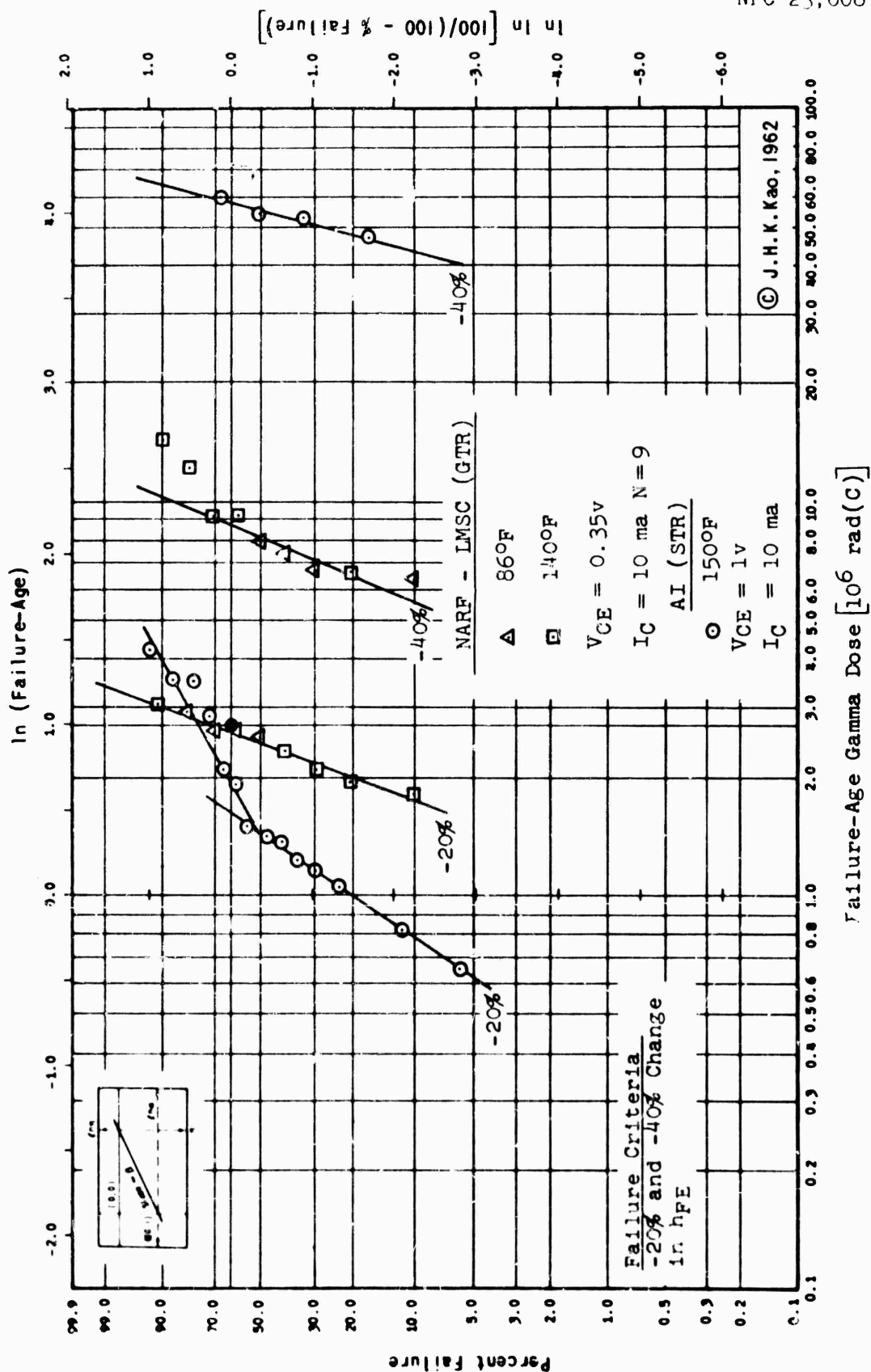


Figure 24 Weibull GTR and Co^{60} Radiation-Reliability Plot of h_{FE} for TI 2N744 Transistors

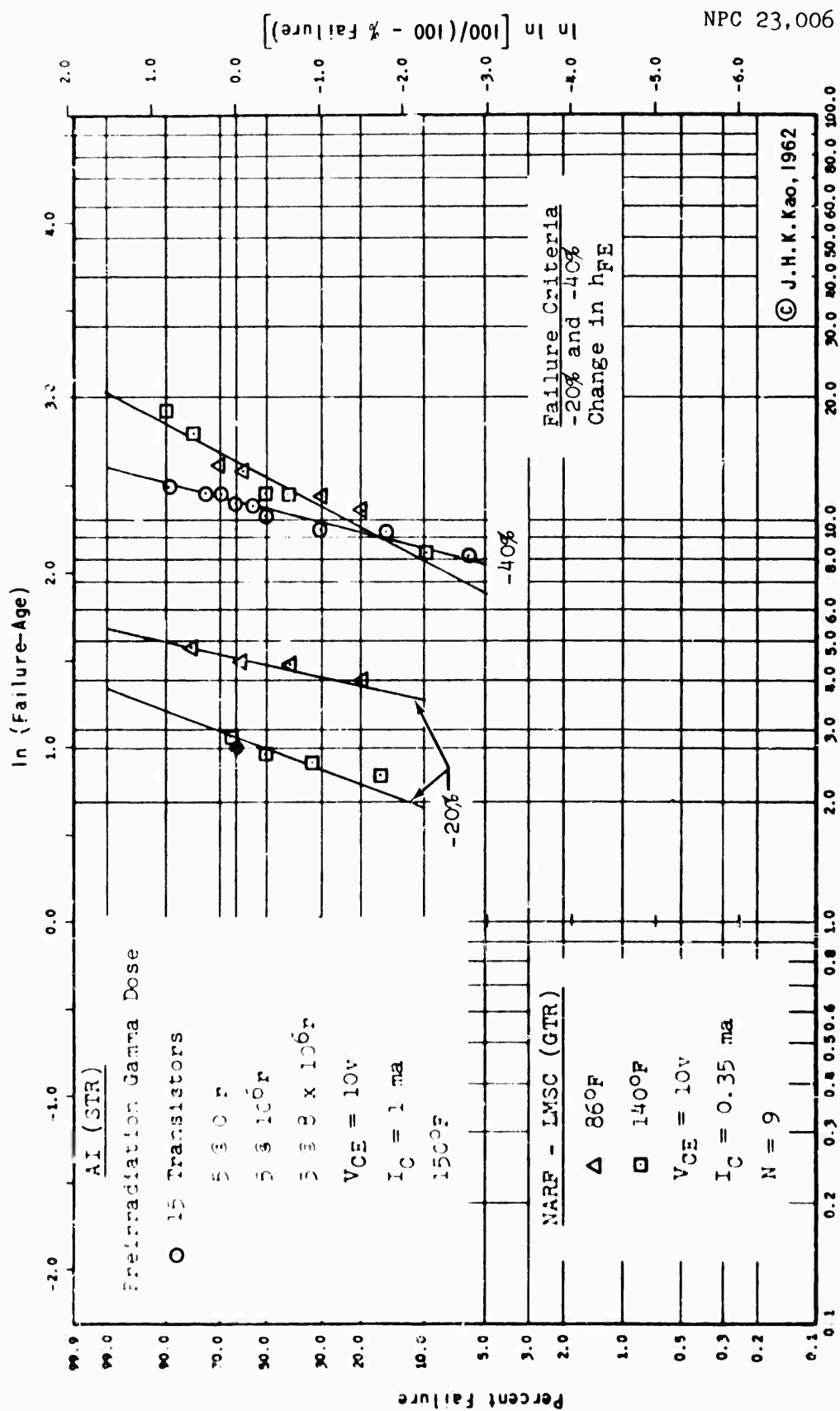


Figure 25 Weibull GTR and STR Radiation-Reliability Plot of h_{FE} for TI 2N744 Transistors

NARF-LMSC (GTR) and AI (STR) radiation temperature data for TI 2N744 transistors.

Although the 86° and 140°F data plotted in Figure 23 appear to be different, the reliability plots of these data show that the difference may not be important from an engineering standpoint; in fact, if these curves represented the true situation, there would be points on the reliability curves where the effects would be the same and then reversed. However, the sample size is not large enough to support this observation.

3. Sensitivity of Operating Conditions

Figures 26 and 27 are additional Weibull plots of h_{FE} for 2N1613 transistors from various manufacturers and for different operating levels of inputs. In Figure 26 the 10-ma data exhibit a greater radiation life-length than the 50-ma data, the difference being a factor of ~ 3 (based on the gamma-dose scale). Because these data were from different tests, the neutron/gamma ratio was slightly different, so that the same data analyzed on a neutron scale showed only a factor-of-2 difference. Rank methods also indicated a significant difference between the groups at different inputs. Figure 27 shows that the 10-ma data have a greater life-length than the 1-ma data by a factor of ~ 6 (gamma-dose scale). This factor is 10 on a neutron scale.

These 2N1613 transistor gain data show the possible variation and sensitivity of component response in a radiation environment for the same transistor type, for different manufacturers, at various levels of inputs, for various scales of measurement, and for different initial values.

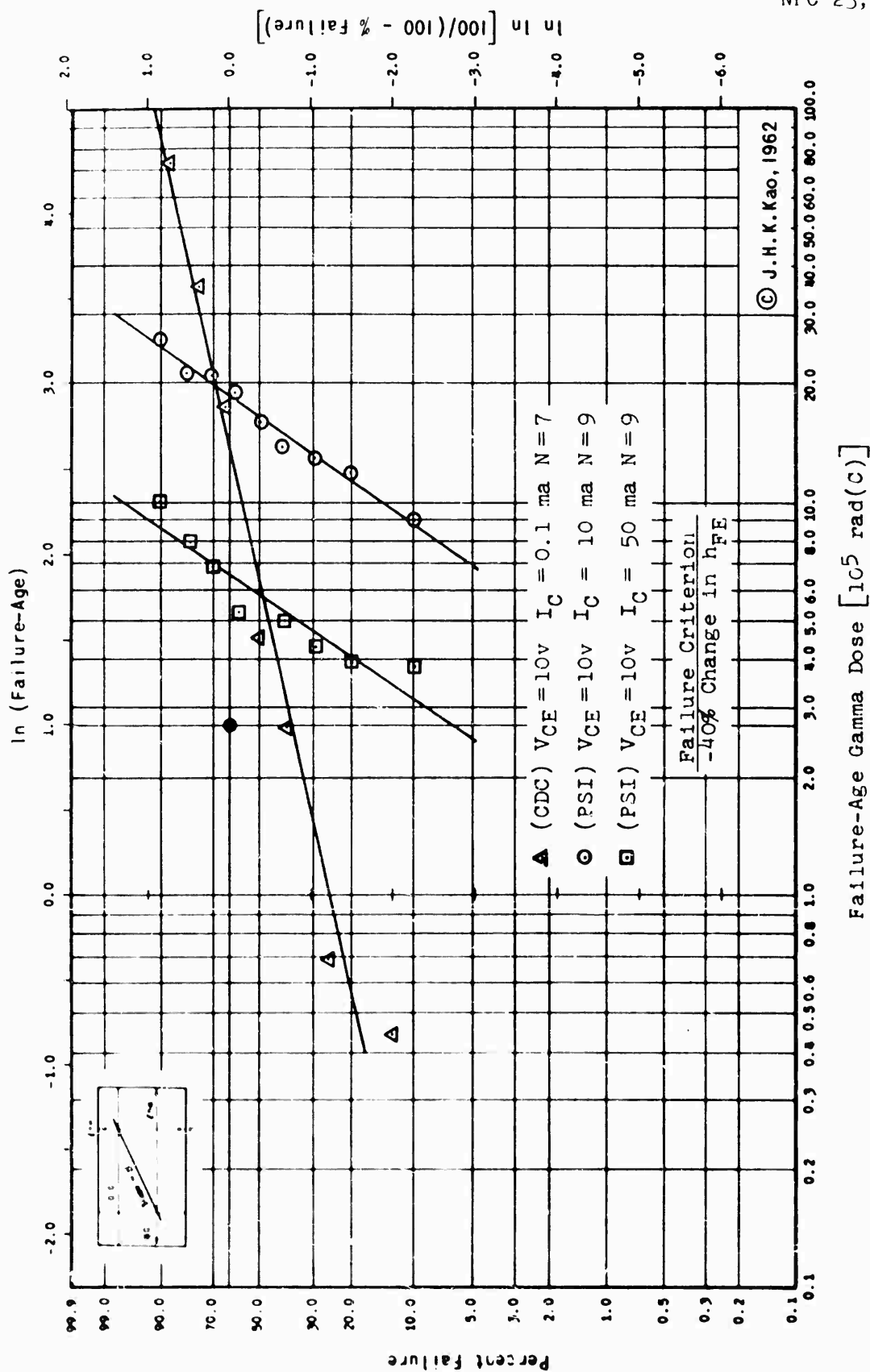


Figure 26 Weibull Radiation-Reliability Plot of h_{FE} for CDC and PSI 2N1613 Transistors

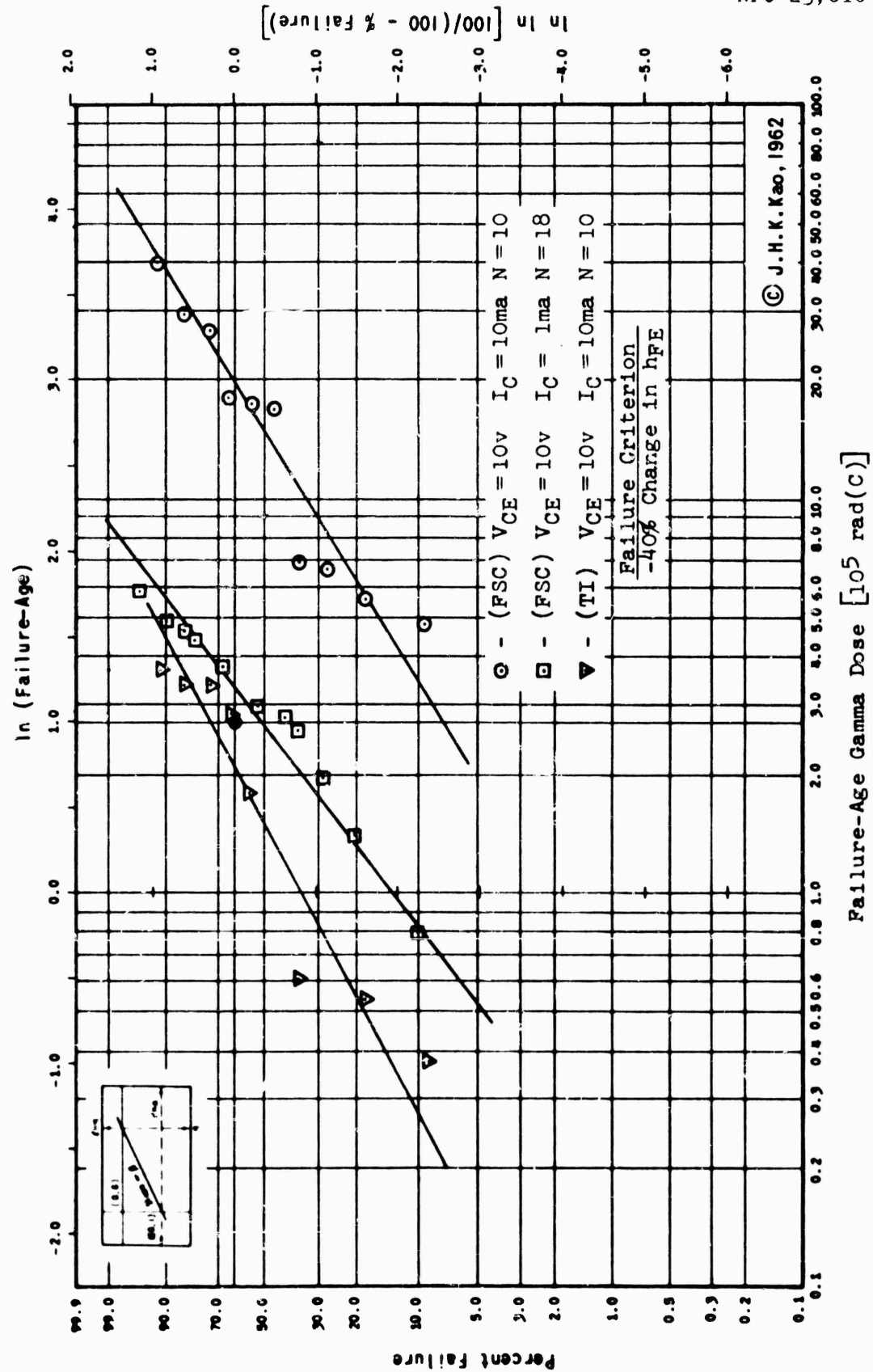


Figure 27 Weibull Radiation-Reliability Plot of h_{FE} for FSC and TI 2N1613 Transistors

4. Radiation Spectrum and Weibull Shape Parameter, β

The reliability curves in Figures 24 and 25 are presented to show the similarity in the slopes (Weibull shape parameter, β) of the curves for reactor and Co^{60} test data for TI 2N744 transistors tested at different facilities and different radiation environments. The temperature data of Figure 23 are combined with some 2N744 Co^{60} Atomics International test data to obtain Figure 24. Figure 25 shows Weibull plots on a neutron scale for NARF-LMSC (GTR) and AI (STR) radiation temperature data for the TI 2N744 transistors. These results are not intended to compare absolute results between facilities, since there are too many differences to be taken into account for a comparison even though the neutron test data are in agreement.

The results indicate that the shape parameter is not a function of spectrum within the limits of the data and the facility characteristics.

5. Probability Distributions

Figure 28 contains Weibull plots of FSC and TI 2N1132 transistor gain. Figure 29 is a log-normal reliability plot of the -40% data in Figure 28 to show the similarity of the graphical representation of different probability distributions that could be used as a failure model (see Section II-1c). Figure 30 is a linear presentation of the -40% FSC data from Figure 28. These data show the similarities in the slope (shape parameter of the Weibull distribution function) for the same transistor type made by different manufacturers.

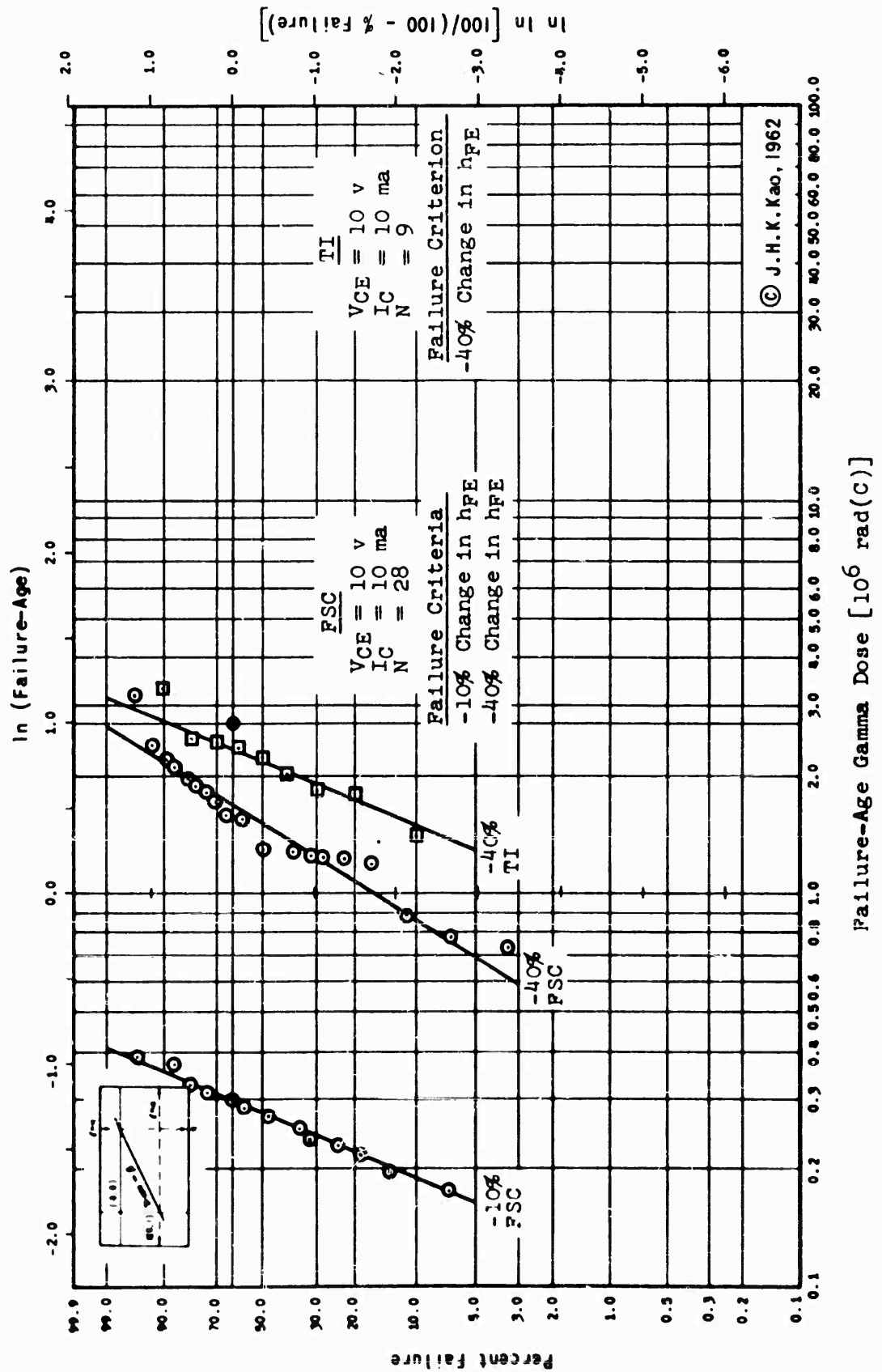


Figure 28 Weibull Radiation-Reliability Plot of h_{FE} for FSC and TI 2N1132 Transistors

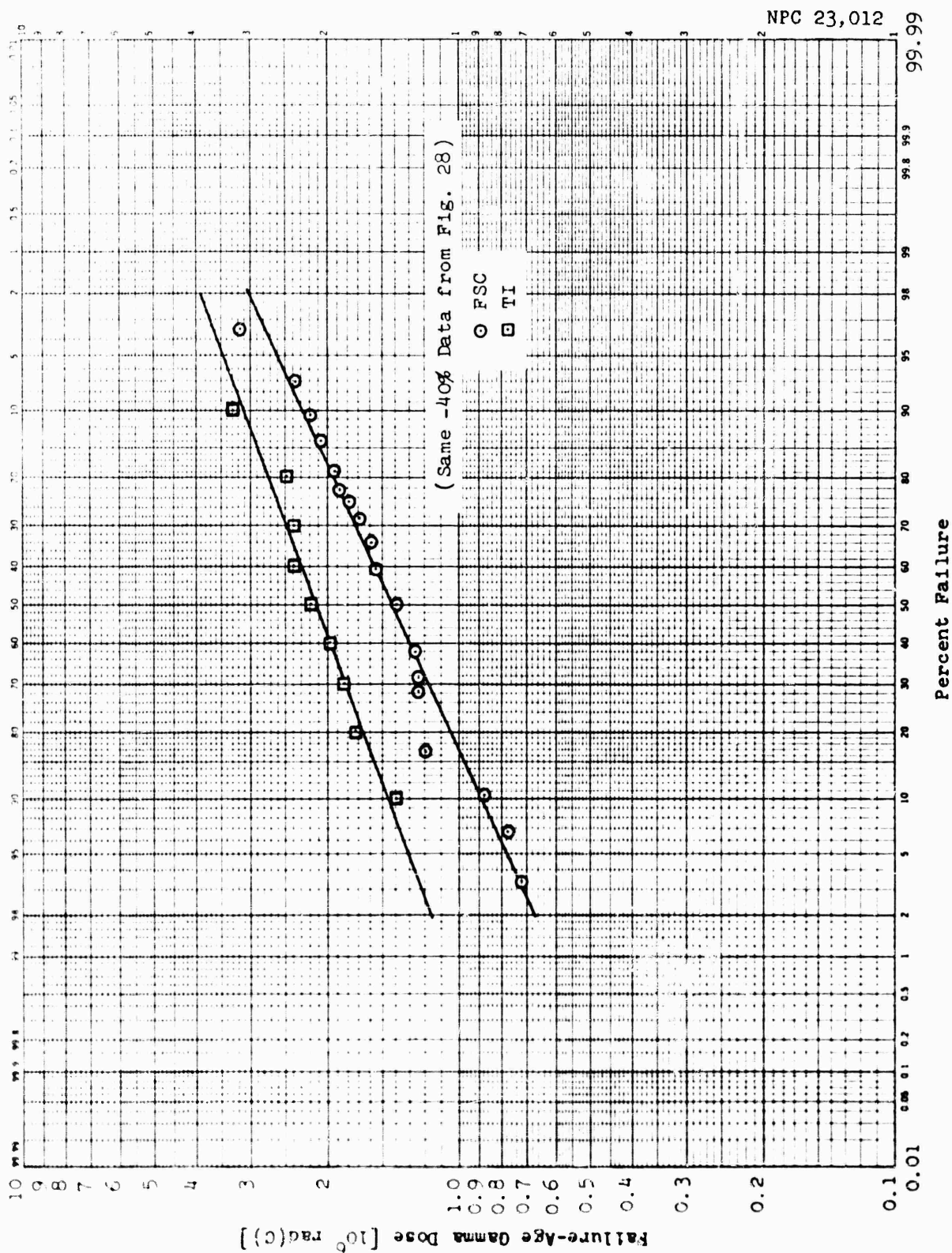


Figure 29 Log-Normal Radiation-Reliability Plot of h_{FE} for FSC and TI 2N1132 Transistors

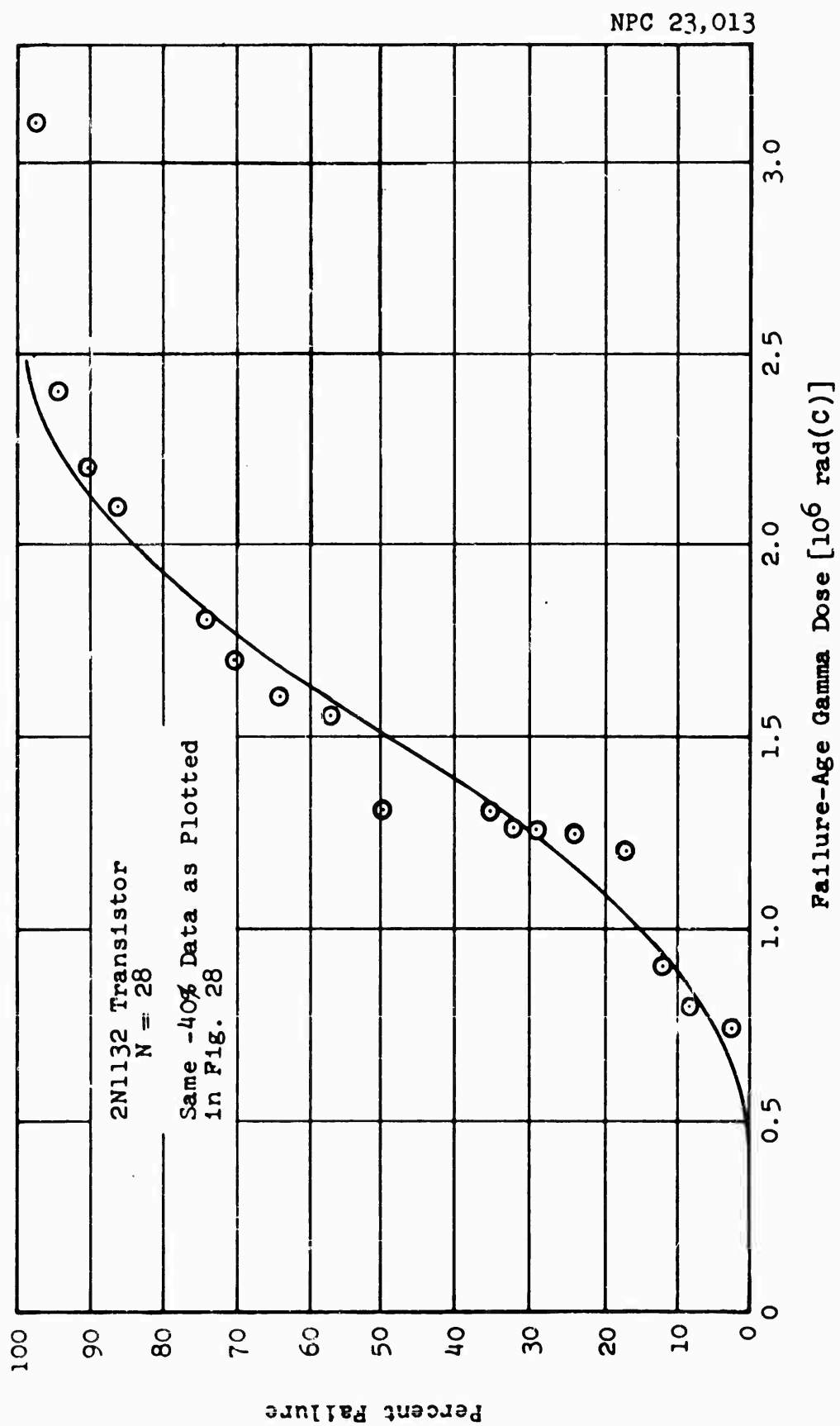


Figure 30 Linear Radiation-Reliability Plot of h_{FE} for
FSC 2N1132 Transistors

6. Failure Criteria

Figure 31 is a Weibull plot of h_{FE} made from data taken during a 1,000-hr life-test of TI 2N917 transistors. These curves show the effect of the different failure criteria: -20% and -40% change from initial value and the manufacturer's specifications of a minimum gain of 9.75 (a pretest minimum limit of 15 and a test limit of 35% less) and an I_{CBO} maximum of 10 na. The failure rate of 1.8%/1000 hr is the quoted estimate based on the exponential assumption. The -20% and -40% curves are the change in h_{FE} taken from the manufacturer's raw data. It is obvious from this presentation and those to follow that analysis of data must be performed using failure criteria that correspond to the designer's use. As has been observed in the radiation data and the life-test data, the slopes of the curves for the percent change failure criterion do not change appreciably when going from 20% to 40%.

Figure 32 is a plot of a 5000-hr sequential life-test for a 2N1717 transistor. The plotted data are based on the manufacturer's failure specification, and the failure rate of 1.6%/1000 hr is the quoted estimate based on the exponential assumption. The original data are plotted at the recording time in one case and at the midpoint of the interval of the recording times in another case. One would not expect the failures to occur just at the recording time, so that the actual failure time is not accurately known. The procedure then is to plot at the midpoint of the interval. As can be seen from the graph, the failure rate for the plotted data is greater than 1.6%/1000 hr at the time of 1,000 hr, and when extrapolated to

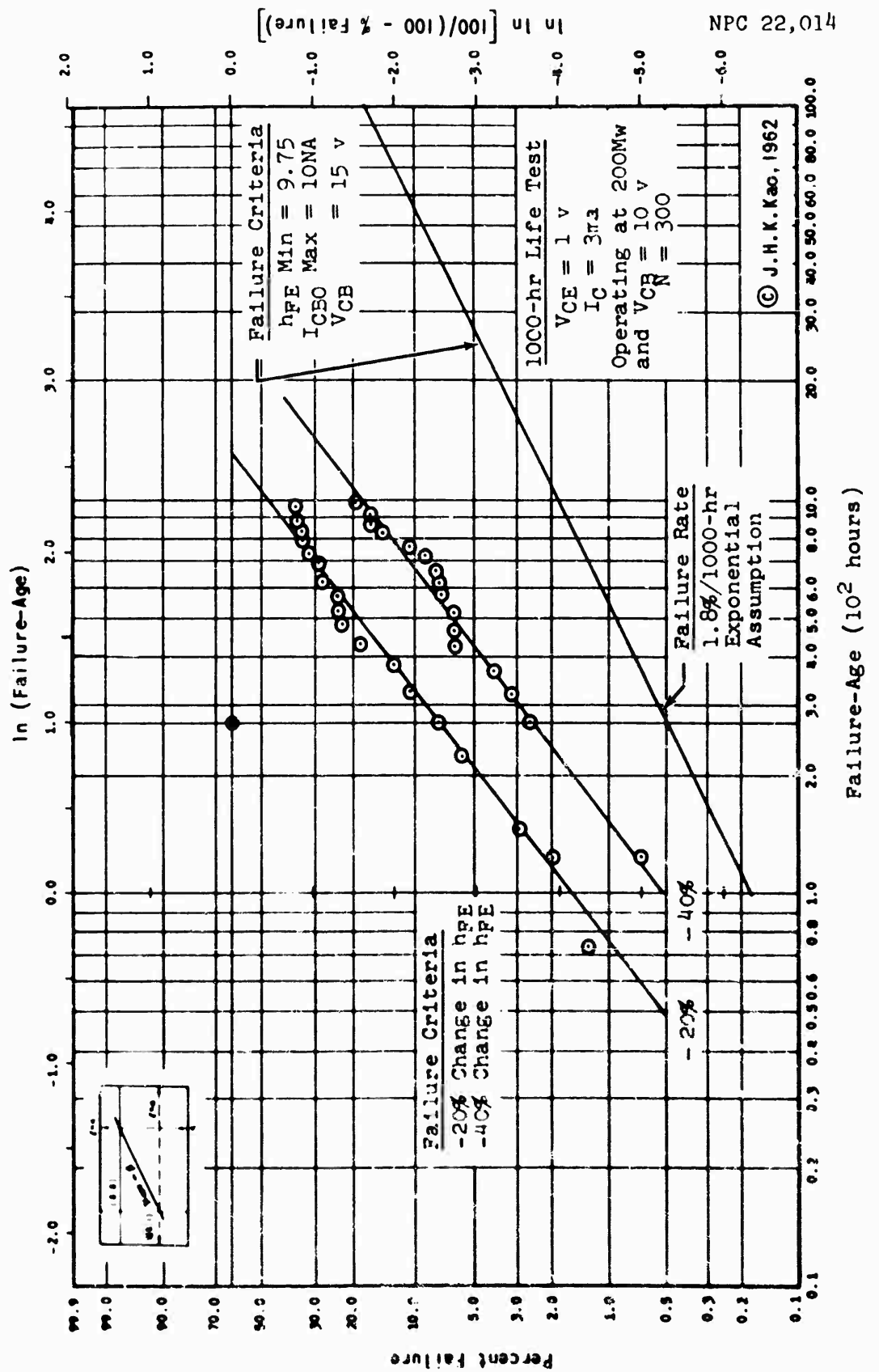


Figure 31 Weibull 1000-Hour Life-Test Reliability Plot of h_{FE} for 2N917 Transistors

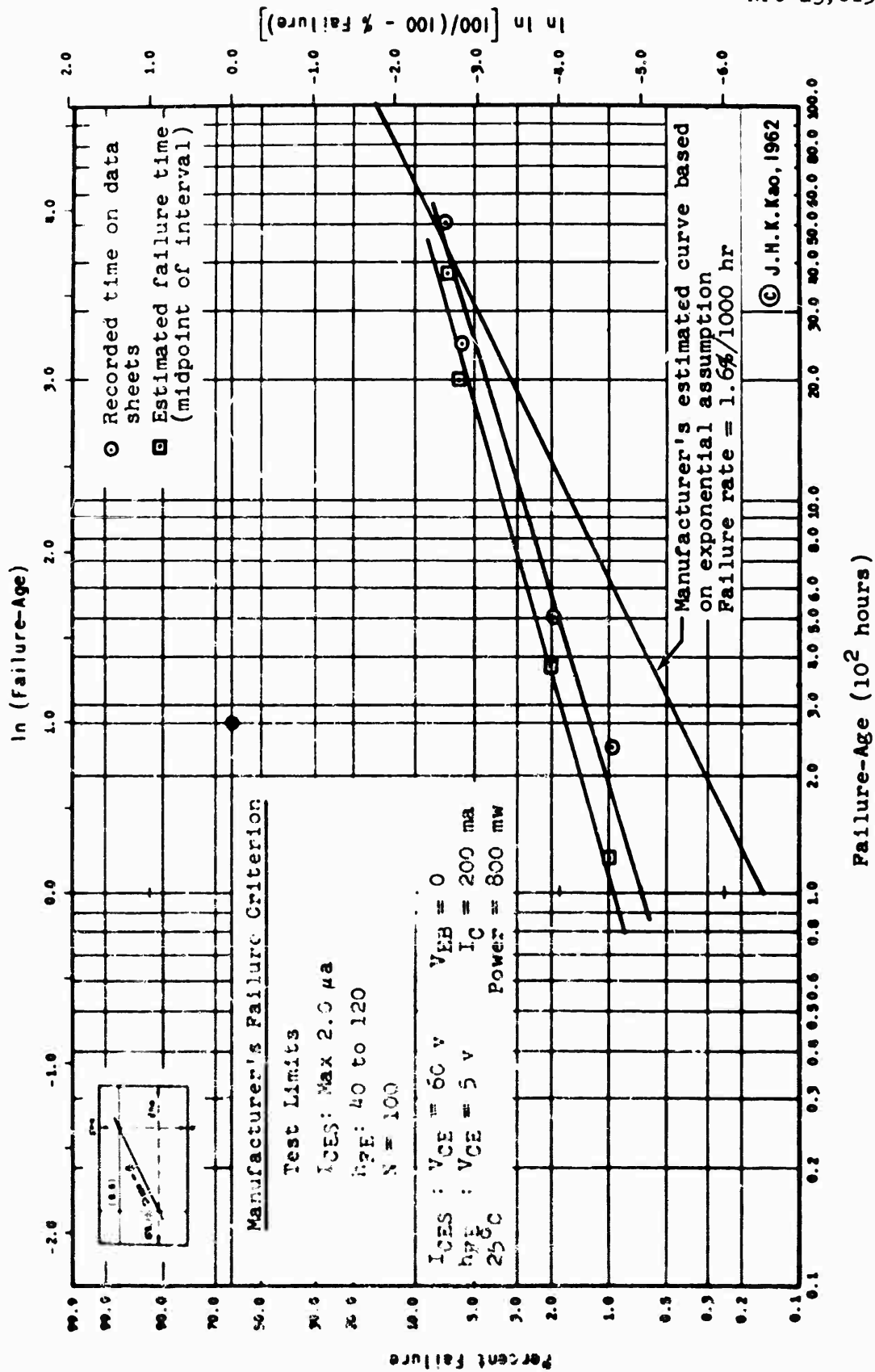


Figure 32 Weibull 5000-Hour Life-Test Reliability Plot of h_{FE} and ICES for TI 2N1717 Transistors

10,000 hr it is approximately 1.6%/1000 hr. In other words, the failure rate for the plotted data indicates a decreasing failure rate with time as compared with the constant failure rate of the exponential distribution.

Figure 33 exhibits the data from an 8000 hr life-test of 300 TI 2N744 transistors. The manufacturer's failure criterion for this set of data is a minimum and maximum h_{FE} for initial and test-limit specifications, and applies to the whole lot. These initial h_{FE} test limits are from 40 to 120, and the final test limits are from 32 to 144. The 10% change criterion, for example, classes a failure for an initial gain of 50 when the gain value reaches or exceeds the limits of $50 \pm 10\%(50)$. This failure criterion is quite severe and applies to the individuals in the lot. There were no catastrophic h_{FE} failures and only one degradation failure for I_{CES} on these 300 transistors. Therefore, the 10% failure criterion was used to provide some "definition" failures to construct a Weibull plot. The manufacturer's estimates based on the exponential distribution assumption are included as dashed lines, namely, a failure curve of 0.042%/1000 hr and a 90% confidence failure curve of 0.16%/1000 hr. The exponential distribution does not appear to apply to the -10% data.

Figure 34 contains the Weibull plots of the forward volts, V_f , for four different types of diodes. These plots are similar to the transistors in that the shape parameters are predominantly greater than one. The initial values for a given type of diode are, in most instances, clustered about a typical value and not

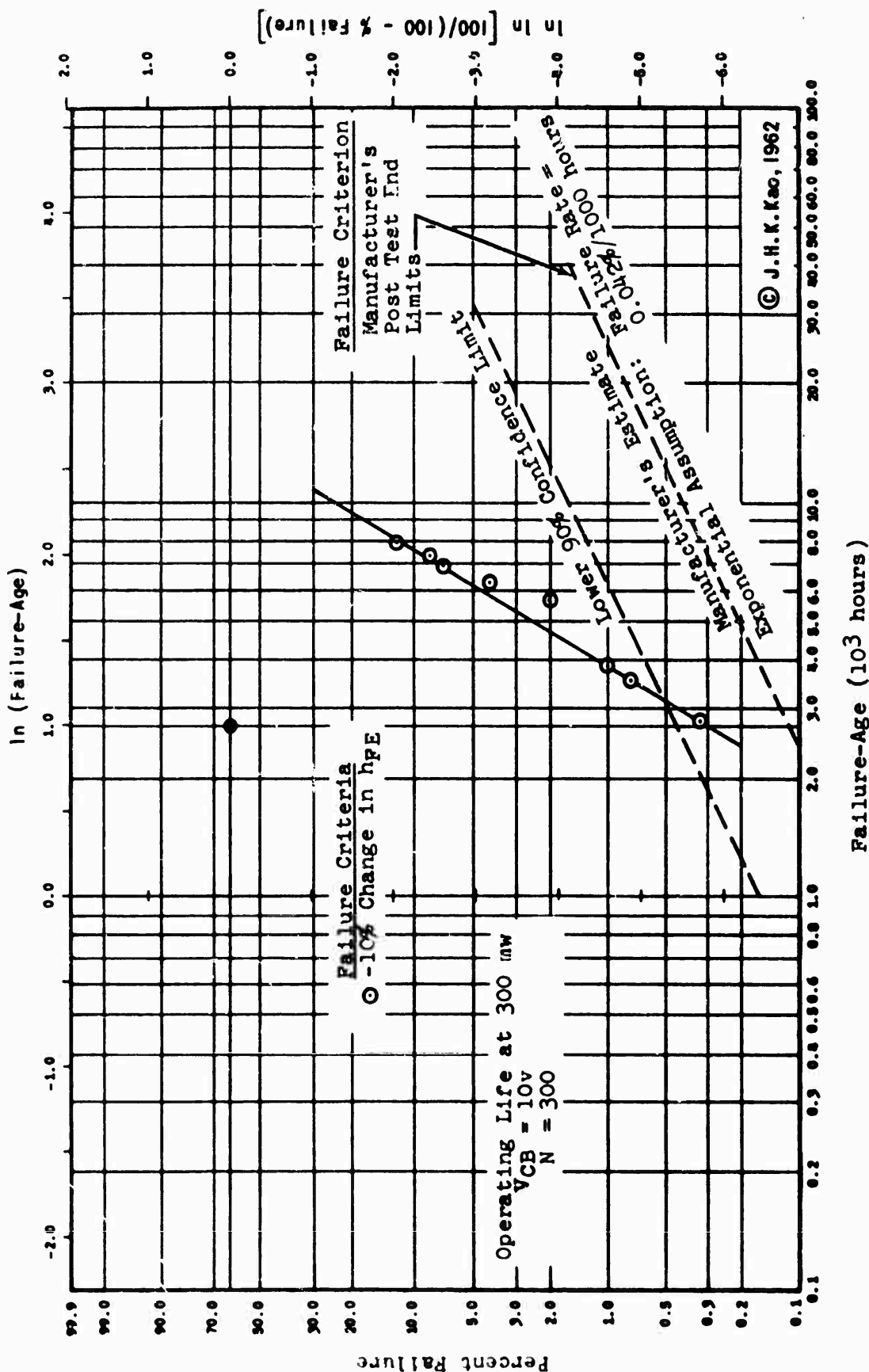


Figure 33 Weibull 8000-Hour Life-Test Reliability Plot of TI 2N744 Transistors

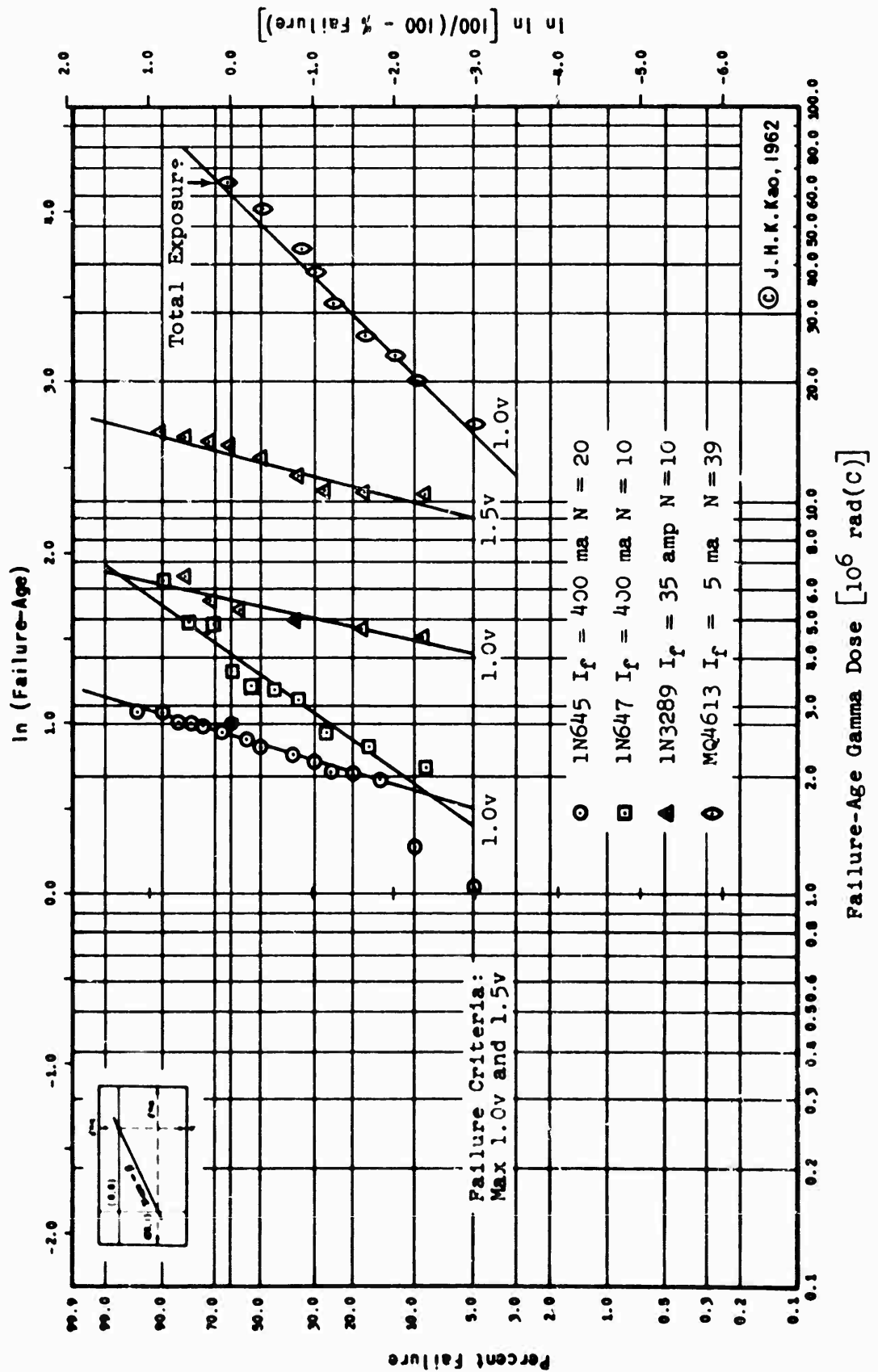
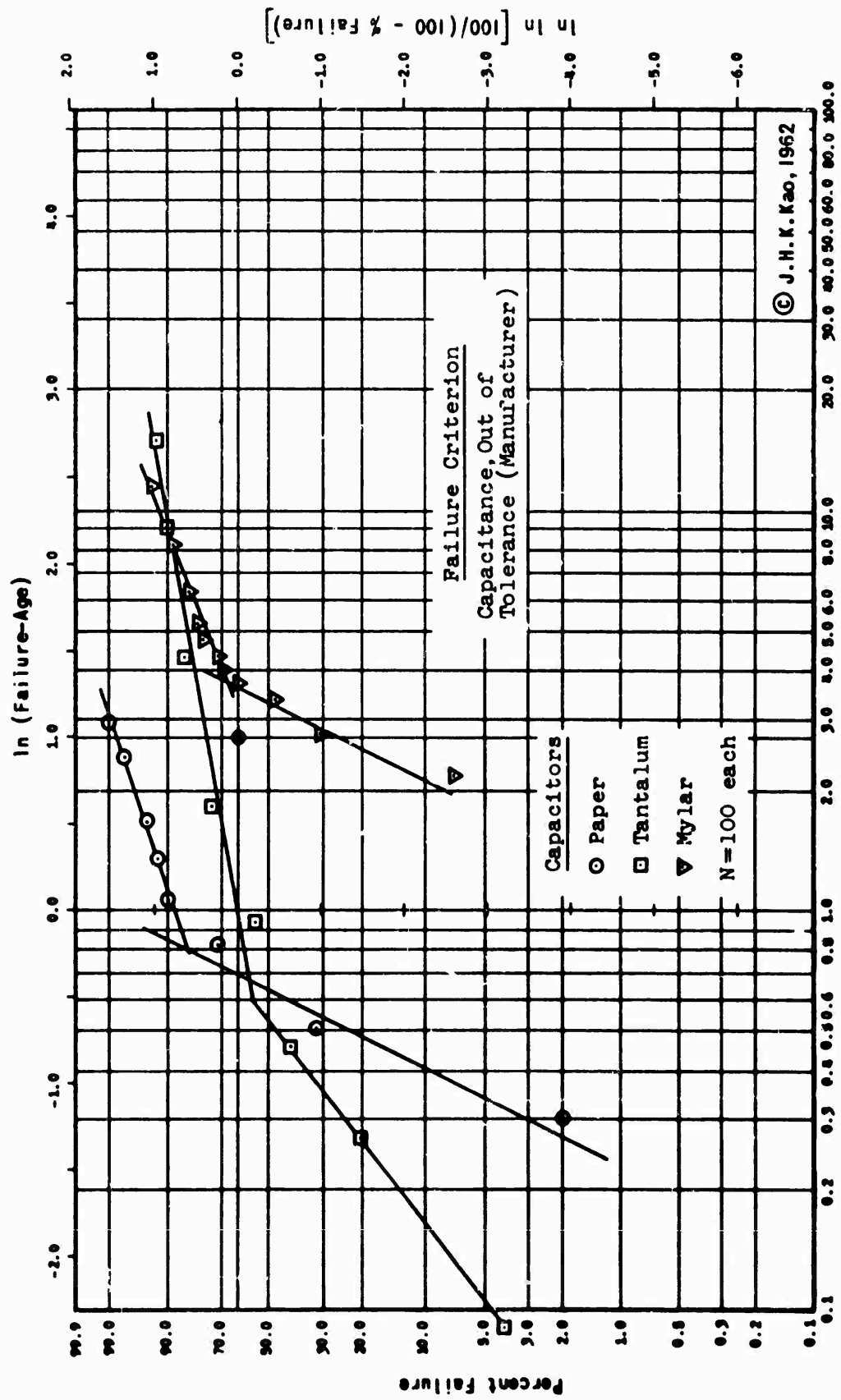


Figure 34 Weibull Radiation-Reliability Plot of Forward Volts for Four Diodes

strewn over the large range as were the gains of transistors. These data are more easily handled in analysis and plotting, and the failure criterion is a maximum value of 1.0v. Other failure criteria considered and plotted in the course of this study were 1.5v and 2.0v. An example of 1.5v is included in Figure 34. It was noted that when an outlier did occur in the initial values, the radiation response was sensitive to the initial value. For example, an initial $V_f = 0.75v$ would have a longer lifetime than a $V_f = 0.8v$ when the failure criterion was a maximum of 1.0v. Not all of the MQ4613 diodes exceeded 1.0v before the end of the test; thus, only 26 out of the 39 units tested are plotted as failures.

7. Bimodal Response

Figure 35 is a Weibull plot of some capacitor radiation data. These data represent a possible bimodal response — at the beginning, a steep slope and, later, a less steep slope. This is probably the result of more than one failure mode operating. An example of this is the case in which the units under test are subject to a high early-hazard rate followed by a lower hazard rate that persists after the weaker units have failed. The percent failure point where the two separate curves for a given capacitor meet can be used as an estimate of the percent of the population of capacitors belonging to each of the observed responses; for example, the two curves of the Mylar capacitor meet at approximately the 64% failure point. An estimate of the proportion belonging to the "weak sister" population is 64% and, to the population which persists, is 36%. As one article put it, if there were some way in which one could make the defective



Failure-Age Neutron Flux ($10^{15}n/cm^2$ for $E > 2.9$ Mev)

Figure 35 Weibull Radiation-Reliability Plot of Capacitance for Paper, Mylar, and Tantalum Capacitors

parts (weak sisters) stand out from the rest when they come from the manufacturer, the total population could then be divided into two groups; those "weak sisters" with some latent defects and those items representing near-perfect components.

8. Conclusions and Discussion

The sensitivity and mathematical models analysis (Section II) was a parallel effort; as information became available in one, it was channeled to the other to see where it would fit in. In some cases, the "random" variation of the data was so great that it was difficult to distinguish a general pattern for a given set of observations. This difficulty can be expected when dealing with small-sample sizes from skewed distributions. However, when the results included several small sets of data which had similar response characteristics, speculation was made concerning general patterns of response.

Not all data considered in the investigation have been presented. Certain data were selected on the basis of their being representative of various arguments, problems, and philosophies associated with radiation-reliability analysis.

Many data were plotted on various probability papers, such as Weibull and log-normal, to examine the application of reliability analysis to radiation-effects response curve data. It is concluded that present-day methods of reliability analysis are applicable to the radiation response curve data. This is not intended to imply that the present-day methods of reliability analysis will solve all problems, but that these methods are as applicable to radiation

reliability as they are to the usual applications. The sample sizes used in the existing data are, in most instances, too small to verify any specific failure distribution as being "best;" but, upon examination of a great quantity of data there is strong evidence to support the conclusion that the shape parameter β of the Weibull distribution is predominantly greater than unity for the radiation-effects response curve data rather than $\beta = 1$, as assumed for the exponential distribution. The life-test data analyzed for this study showed that (when sufficient sample sizes were available) the shape parameter β of the Weibull distribution was predominantly less than unity when based on the manufacturer's failure criterion. This same conclusion was reached by ARINC Radio Corporation for the data they analyzed (Ref. 14). Therefore, in radiation and life-test reliability analyses, it is recommended that any assumptions that are made be verified by test data rather than using the thumb-rule of the exponential distribution. This approach will require that more effort be directed towards the selection of failure models to arrive at a more realistic approximation to the existing situation.

In this search for a more realistic approximation, there are certain parameters and environments that need to be considered. Although some of the effects of environments and parameters are discussed elsewhere in the report, a brief summary follows. For the radiation rates and times for the existing data that have been analyzed, it is quite obvious that radiation is the most degrading environment. Most radiation failures, as well as life-test failures, are of the degradation type (response curve) that are defined as failures by specifying a failure criterion. The choice of failure

criterion, as well as the scale of measurement, affects the life-length of the device considered. In much of the data analyzed the shape parameter β of the Weibull failure distribution did not change appreciably when the failure criteria, the operating level, the temperature, and the radiation source were changed. This indicates that the form of the distribution remains fairly constant, the difference in displacement being the result of a change in the scale and location parameters. A constant-shape parameter is almost a must if one is to estimate environmental effects and relationships by reliability analysis. It is difficult enough with skewed distributions of this type to estimate and compare environmental effects within a given type of failure distribution — let alone trying to make these comparisons when the distributions themselves are changing.

The effect of operating input levels has been observed to increase the life-length of a component in a radiation environment in some cases and decrease it in others. The gamma Co⁶⁰ radiations comparing active and passive components show that the h_{FE} lifetime of the active device is greater than that of a passive device. In a neutron radiation, these effects were not observed. When temperature effects were observed in radiation data, the tendency was to increase the h_{FE} lifetime at the higher temperature.

SECTION IV

QUALIFICATION OF RADIATION-EFFECTS RELIABILITY TECHNIQUES

The first subsection to follow outlines a program to qualify and refine the mathematical models developed in Section II. This project will also evaluate radiation-hardening techniques that will be used in the qualification score of the program. The second subsection discusses the research and development that will be required to establish the interaction between the nuclear and non-nuclear environments associated with NAP.

1. Qualification of Mathematical Models

The present investigation into radiation reliability, with respect to the analysis of radiation-effects response curves and life-test data, indicates that most failures of components are the result of out-of-tolerance failures and are classified as degradation failures when a certain limit is exceeded. These data are predominantly of the response curve type, in which the parameter of interest is changing as a function of time and/or radiation exposure. In the reliability analysis of these data, a range of failure criteria is specified and the time and/or radiation exposure of components is recorded at different failure points. The cumulative frequencies are then plotted to ascertain the nature of the failure distribution for the different failure criteria specified. A transformation is rarely made from variable (response curve data) to a discrete value. Sensitivity is lost in the transformation for detecting the differences between conditions being studied, whether they be radiation, time, temperature, or combinations of conditions.

In the present study, the transformation from response to binary has been performed so that the radiation data can be analyzed from a radiation-reliability approach.

Even though components exhibit a continuous variable type response, they are often used in circuits whose outputs and operation are of a binary nature or a Go - No-Go response, such as gates and flip-flops. Thus, some circuits make the transformation from continuous to discrete values by the very nature of their operation. In those circuit types, the response will appear as a catastrophic failure even though the cause is a gradual change in one or more of its components or some combination thereof. Each type of circuit will not fail at exactly the same time, radiation exposure, and/or component values. Therefore, each type of circuit will have failure distributions of its own.

These circuit failure distributions will be a function of the component parts. A very critical component with poor reliability, as compared with the other components, will dominate the failure and will be easily recognized. When all the components of a circuit have similar and closely related failure distributions, their combined failure mode will be difficult to ascertain.

On the basis of the above considerations, a test philosophy based on testing at the circuit level with component monitoring is proposed. This test philosophy is based on the development of suitable models at the circuit level with component data inputs, in order to describe or approximate the observed circuit response over the region of application expected. The development of these

models is predicted on the use of past data, components tests, theory, and experience. The approach to this development is done by an iterative procedure in which the models are tentatively entertained (not assumed), strained in various ways over the region of expected application, and freed of their defects. The nature of the defects, in conjunction with the experimenter's technical knowledge, is used to suggest changes and remedies leading to new models which, in turn, are tentatively entertained and submitted to a similar straining process. This process goes on until one is satisfied that suitable models have been found.

The test program is initiated by choosing available components, circuits, and systems that are of proven reliability in and out of a radiation environment and are compatible with the expected application. A flow diagram of the overall qualification program is shown in Figure 36. Whenever possible, the selection of these devices is made on the basis of successful prior use in related applications. A 3-stage analytical analysis and development program is then performed on the selected devices, as described below.

In Stage I, an analytical analysis of these selected devices is made to predict whether off-the-shelf devices will operate satisfactorily in the region of application.

In Stage II, off-the-shelf devices that are not acceptable are radiation-reliability-hardened by substitution of radiation-tolerant and reliable components. A similar analysis to that in Stage I is then made on these "hardened" devices to predict their operation in the expected application.

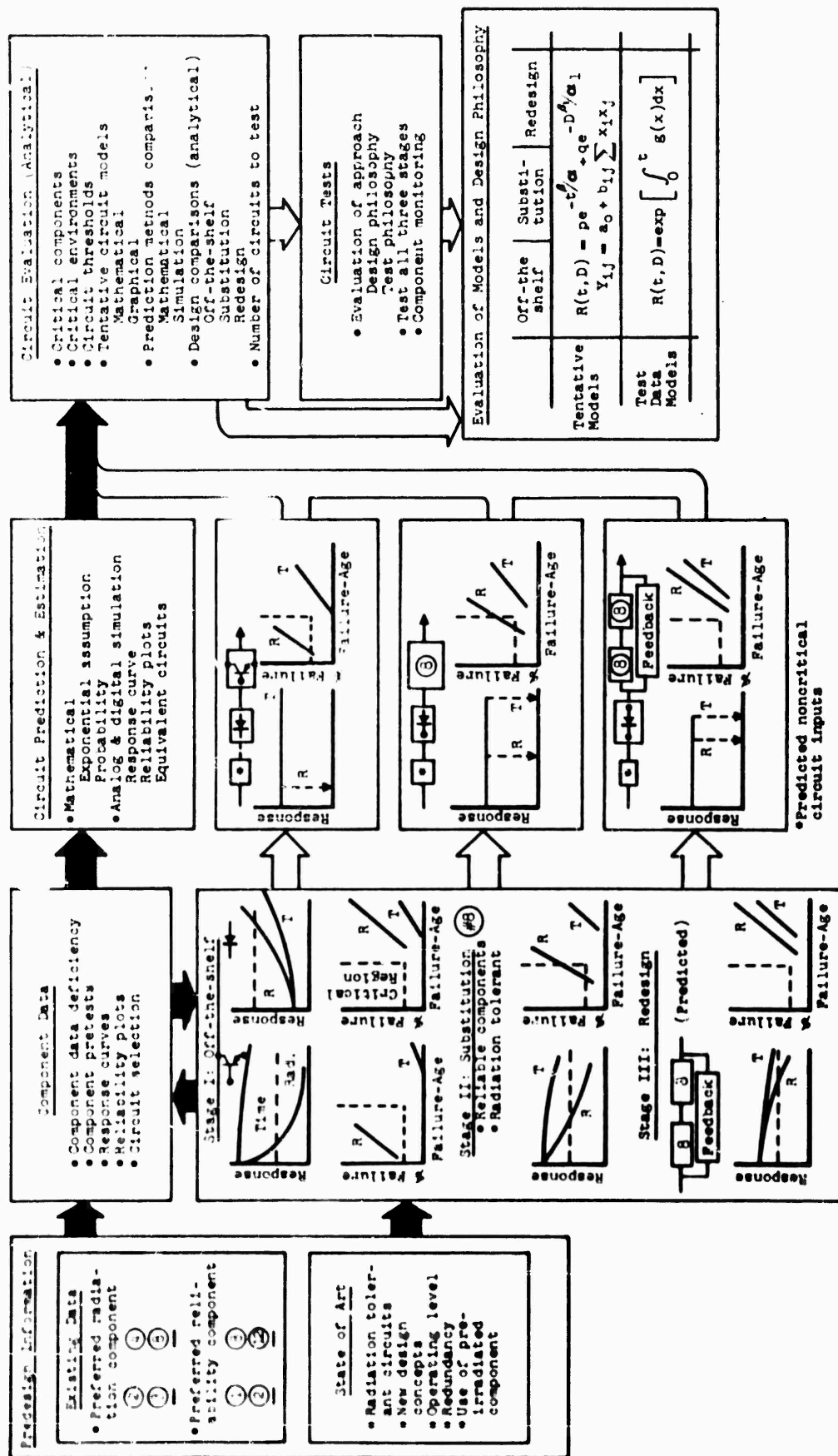


Figure 36 Qualification of Mathematical Models

In Stage III, the circuits and systems that are not acceptable in the previous two stages are channeled into redesign, which may be of various forms or combinations of the following.

- . Redesign of the basic device around predictable response curve data
- . Power-rating of components
- . Redundancy
- . Use of preirradiation components

A preliminary test program at the component level will be required to obtain accurate data under the conditions that will be used in qualifying tests. After obtaining data on the preliminary components, the circuits and their model predictions are refined and then a radiation test is conducted on the circuits.

The test data are compared with the predicted models to ascertain the quality of the prediction methods. If the methods prove successful, these methods should be capable of predicting the reliability of circuits in a long-term radiation environment.

In addition to evaluating and refining mathematical models, the success of methods of radiation-hardening used in this program will be determined.

2. Problems of Research and Development to Establish NAP Test Techniques

One of the initial steps in this study was the review of the basic philosophies of other programs to determine their applicability to utilization of Nuclear Auxiliary Power (NAP). The first subsection to follow discusses the LASV-N, Telstar I, and SNAP 10-A programs.

The second subsection describes the matrix of the conditions involved in long-term nuclear missions, then sets forth the major problem areas relative to the utilization of NAP.

a. Philosophies of Other Programs

The basic philosophies of other programs were reviewed and evaluated for their applicability to the utilization of NAP. These programs were time-limited and little effort was applied to methods of predicting probable lifetime. In addition, the operation times of these missions were short compared to the minimum NAP requirement of about 10,000 hr. Consequently, the results of these programs are not directly applicable as far as radiation reliability is concerned.

The reliability or test philosophies of LASV-N, Telstar I, and SNAP-10A and their disadvantages as applied to long-term missions are discussed below.

(1) LASV-N

The LASV-N development was confronted with environmental problems that included high values of temperature, vibration, and nuclear radiation. The approach to reliable design consisted of defining the subsystem concept; establishing the environmental and mission requirements; designing the subsystem, using the most reliable components available (based on state-of-the-art data) for the environment in which they were to function; and then testing the subsystems in a nuclear environment. Any subsystem that would not stand the test to a prespecified nuclear exposure level would be modified by replacing the weakest components with those of higher radiation resistance (Ref. 21). The modifications and tests would continue

with the design goal established as a point where three or four test articles operate successfully when exposed to a time-integrated energy flux or dose 100 times more severe than operational conditions. This successful operation would demonstrate "sufficient reliability." This philosophy is contingent on the argument that the exponential failure law is the most conservative and realistic assumption.

This "conservative" test philosophy may result in the rejection of good items, especially considering semiconductor devices which may be much more reliable in the two-decade-lower mission environment as opposed to, say, ceramic tubes. An advantage of this test philosophy is the low probability of accepting radiation sensitive components.

Testing two orders of magnitude in dose for a 10,000-hr mission would result in unrealistic reliability predictions.

(2) Telstar I

Another design philosophy which is pertinent to space system operation, and under which consideration was given to radiation exposure, is that of the Telstar I. The Telstar I project was completed in 15 months, from the start of the program to launch (Ref. 22). The short development period imposed restraints on a reliability testing program, thereby calling for the use of components of proven integrity in the satellite system. In order to acquire components of proven integrity, the component selection program was based essentially on that originated for submarine cable devices.

This selection program consisted of:

1. Design of the component for the required environment
2. Careful control of manufacturing processes
3. Elimination of potential early failures by screening tests
4. Selection of the most stable components
5. Simplest design to perform the function intended in the anticipated environment

The principles and techniques used in the Telstar development program are applied either totally or in part to the development of many complex missile and space systems. This approach has produced fairly successful missions because of the availability of reliable components. However, due to the uncertainty of the radiation reliability of components, the Telstar approach cannot be applied to long-term nuclear missions until appropriately qualified long-term materials and components are available. In addition, estimated reliability, as in Telstar, could only be calculated using each component reliability value, not with the measured reliability of a subsystem or complete system. A reliability estimate using component values is naturally more uncertain than one using subsystem or system values.

(3) SNAP-10A

Another program of interest that resulted in the need to develop radiation-tolerant systems was the SNAP-10A Program. The aims of program were (1) to obtain sufficient information to successfully accomplish 1-year ($\sim 10,000$ -hr) missions using SNAP units, (2) to attempt to relate system and component failure threshold and predict mean-time-to-failure for parts and systems, and (3) to obtain a high degree of reliability for flights in the most economical and

timely manner consistent with good engineering practice (Ref. 23).

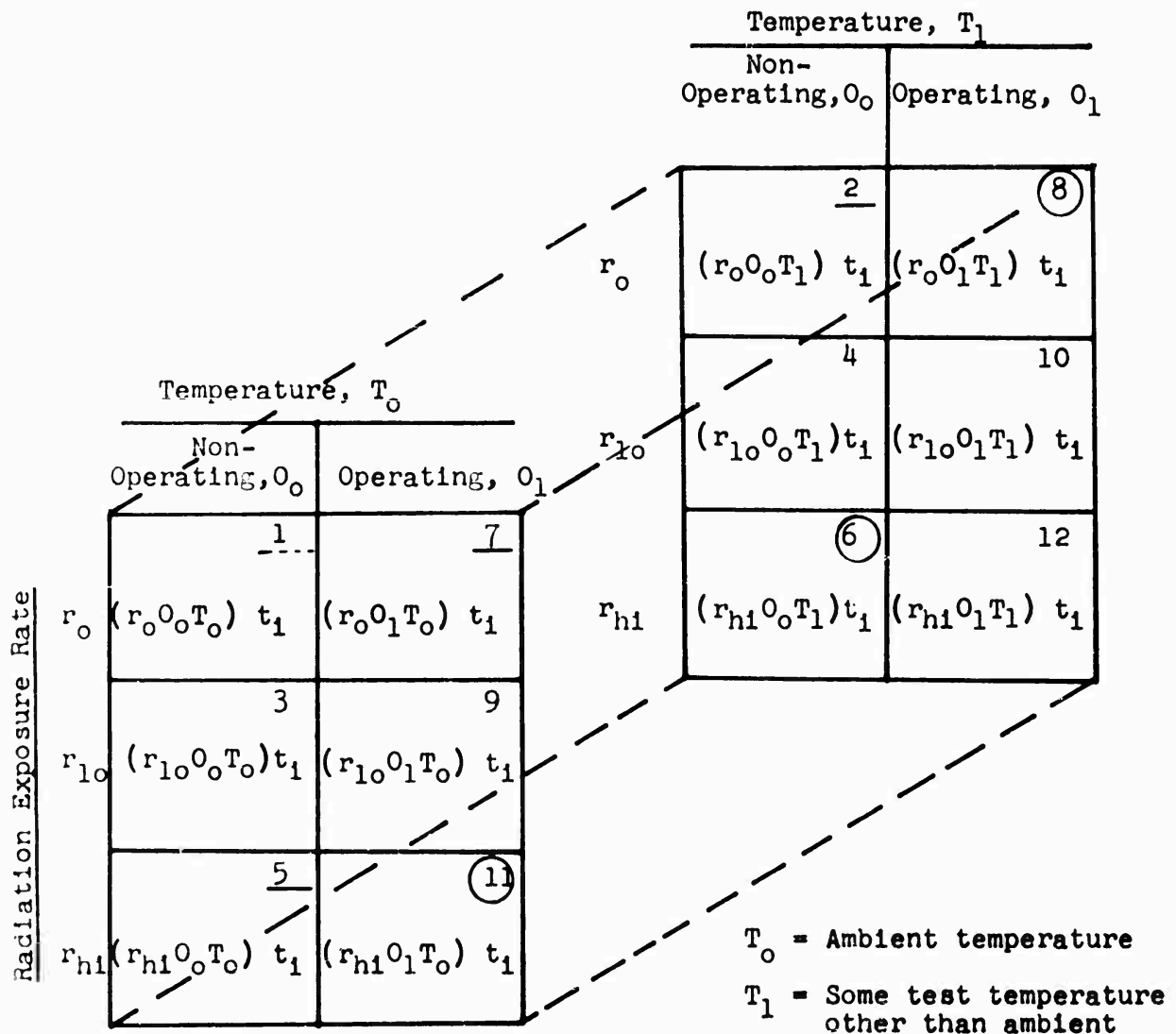
The aims of the program were not fully realized because program schedule and budget problems would not allow sufficient testing for accurate predictions of equipment reliability. Therefore, the decision was made to exclude radiation-effects considerations from reliability predictions. Instead, a series of 100-hr tests was performed to pick radiation-tolerant components and systems with unknown qualification on the reliability of the mission.

b. Major Problem Areas

(1) Relationship Between Environments

As stated above, most programs have been performed with a "test" philosophy, the environments have been simulated to match the mission, and little emphasis has been placed on determining the relationship between the various environments. One exception to this case is operating level, where the response of some components is established as a function of operating level. It is common practice to derate components to increase reliability. In some cases, however, available data (limited to short-term tests) show that operating conditions enhance the radiation lifetime of a component when the nuclear environment is added. This example demonstrates the obviously complex nature of combined nuclear environments and the difficulties of long-term reliability estimation — the basic reasons for the requirement of establishing detailed test techniques before practical development of nuclear auxiliary power can be realized.

Figure 37 is a matrix representation of a 2 x 2 x 3 factorial arrangement of the environmental conditions (referred to as



- r_0 = Zero nuclear exposure rate, i.e., no radiation
- r_{10} = Low nuclear exposure rate applicable to NAP application
- r_{hi} = High nuclear exposure rate with respect to NAP application
- ($r_0O_0T_0$) = Notation used to indicate the combination of environmental conditions of no radiation rate, r_0 , nonoperating, O_0 and ambient temperature, T_0
- t_1 = Time variable, where $i = 0, 1, \dots, h$ hours measured from start of test to determine response curves. A static test will record time at t_0 and post-test, t_n

Figure 37 Matrix Presentation of Radiation Rate and Temperature

"treatments") for estimating the relationship between radiation, operation, and temperature plus time. This presentation is used to show the scope of the problem of determining the relationship of radiation effects and reliability in conjunction with various combinations of environments.

These environments were chosen because they are the predominant conditions that are represented in available data. The notation used in the discussion is listed in Figure 37. The bulk of the data used is for the treatments in Blocks 2, 5, and 7. Blocks 2 and 7 are manufacturer's life-tests and 5 is radiation effects. There is only a limited amount of test data for the treatments in Blocks 6, 8, and 11. No known data exist for Blocks 3, 4, 9, 10, and 12. Based on manufacturer's life-tests, the treatment, $r_0 O_0 T_0$ in Block 1 is assumed to have no degradation effect on the components considered, especially for comparison to typically short-term radiation life-tests. To estimate the relationship between the various environments requires certain comparisons to be made.

Comparing the results of $r_0 O_0 T_0$ and $r_0 O_1 T_0$ will give an estimate of the main effect of going from a nonoperating condition, O_0 , to an operating condition, O_1 . Similarly, comparisons between Blocks 1 and 5 give an estimate of the radiation effect, r_0 to r_{h1} . For Blocks 1 and 2, an estimate of the temperature effect, T_0 to T_1 , can be made. The above are classified as main effects in which only one environment has been changed while the other environments are held constant. In treatments $r_{h1} O_0 T_0$ and $r_0 O_1 T_0$ for Blocks 5 and 7, two environments have been changed so that any observed effect between these treatments could be the result of either r_{h1} or O_1 or both.

A diagonal pair of treatments, say Blocks 5 and 8, show that all environments have been changed. An analysis of these various treatment combinations is used to estimate the 2- and 3-factor interactions. These estimates of the interaction effects are used to determine the relationship (dependence or independence) between the environments. If it can be assumed that the effects are independent, then an estimate of Block 11, treatment $r_{hi} O_1 T_0$, can be made by adding the estimated effect from treatments between Blocks 1 and 7 to the value observed in Block 5. Existing data show that for transistor gain h_{FE} , the main effects of operating, $r_0 O_0 T_0$ to $r_0 O_1 T_0$, are to decrease the gain and increase the failure rate from a negligible amount (0%/1000 hr) at O_0 to a significant amount of (1.8%/1000 hr) at O_1 . From radiation experiments, it is known that the gain decreases as a function of radiation exposure; therefore, the additive effects (assumption of independence) of O_1 , when applied to $r_{hi} O_0$, should give a gain less than that at $r_{hi} O_0$. Some data that have been obtained at Block 11, $r_{hi} O_1 T_0$, indicate that the effect is something less than that at Block 5 - which contradicts the independence assumption.

This same general conclusion holds for estimating effects between Blocks 1 and 2 and addition to Block 5 to estimate \bar{C} . In this case, past data show that the higher temperature tends to increase the h_{FE} life-length under the condition of $r_{hi} O_0 T_1$, as compared with $r_{hi} O_0 T_0$, Blocks 5 and 6, respectively. Again the independence assumption is contradicted.

The results of some special radiation experiments show that there are rate-dependent permanent effects in electronic components such

that a linear extrapolation (acceleration factor assumed 1 to 1) - based on the high radiation rate, r_{h1} , data - to the effects at the lower rate, r_{l0} , will be in error for the same total radiation exposure, E_t . These limited data on rate-dependent permanent effects are not sufficient to cover any broad areas, nor are they sufficient to prove or disprove whether this linear extrapolation is valid or not at the anticipated low rate, r_{l0} , for NAP application. Existing radiation data have been obtained at rates some 100 to 300 times greater than expected for NAP application. Rate studies have been conducted at a factor of approximately 5 difference. Thus, linear extrapolation of rate, independence of temperature effects, and independence of operating effects are not justified by the existing data.

The statements made with respect to the relationship between radiation effects and reliability in conjunction with various environments have been based upon a very limited amount of data and a large amount of intuitive speculation.

The foregoing discussion has pointed out the use of existing radiation effects and reliability data in the determination of environmental relationships. Extension to more environments is a straightforward expansion of the matrix.

A program conducted to determine all the possible relationships between the environments would be an enormous, if not impossible, task and it will be necessary to limit the environments and their variation by careful prediction of the environments associated with utilization of nuclear auxiliary power.

(2) Problems of NAP

The characteristic of NAP that is the basic difference from past nuclear programs is the mission time, a minimum of 10,000 hr. This long life allows the life-length changes to become significant in comparison to nuclear-induced changes. In past short-term missions the nuclear environment has been predominant to the extent that life-length changes are insignificant.

The life-length changes are not simply additive, but are complex interactions between radiation, operating conditions, temperature, time, and possibly other environments. The following section discusses these interactions as represented by available data. The conclusions drawn are that there are not sufficient analytical data nor fundamental knowledge to presently establish the interactions of the environments of a long-term nuclear mission, and it is not feasible to perform a series of long-term tests to develop the systems. Thus, it will be necessary to establish the analytical data and fundamental knowledge that can be used to perform a minimum yet complete development of NAP systems.

The main objective of such a program would be to establish the minimum test duration that can be used and still accurately extrapolate to long-term radiation environments.

Such a program would be initiated by a study to establish the limits of requirements to perform a NAP mission. This initial study would define (1) environmental envelopes, (2) systems and their operation requirements, and (3) representative materials and devices.

The necessary research and development to establish NAP test techniques can be formulated using qualified analysis methods from a program such as that described in Section IV-1 and limited by the NAP requirements.

The solution to these test techniques will require more than test data on a matrix of variations on the range of environments. Fundamental knowledge, such as the nature of material changes in relation to device response and the effects of construction technique, will have to be considered.

SECTION V

STANDARDIZED TESTING AND DATA-REPORTING PROCEDURES

This section is divided into three subsections. The first discusses the planning of an experiment and gives detailed examples that illustrate the importance of trying to predict what data will be obtained and how these data will fulfill the objectives. Although such a discussion may seem exceptionally trite, a cursory search of the literature will show that many studies having poor results or results that are more or less negative would not have been performed if the experimenter had tried to predict the outcome of his test or if he had pre-determined the possible use of the data.

The second subsection suggests some standard data presentation methods, while the third suggests some military specifications in relation to long-term nuclear missions.

1. Planning Experiments

In the initial stages of experiments too little time and effort is put into the planning. This planning stage is at least as important as any other task in the experimental program. The statistician, who expects that his contribution to the planning will involve some technical matter in statistical theory, finds repeatedly that he makes a much more valuable contribution simply by getting the investigator (1) to explain clearly why he is doing the experiment; (2) to justify the experimental conditions whose effects he proposes to compare; and (3) to defend his claim that the completed experiment will

enable its objectives to be realized.

In the initial steps in the planning of a test program it is good practice to make a written draft of the proposal for the experiment. In general, this draft will have three parts: (1) a statement of the objectives; (2) a description of the experiment, covering such matters as the experimental conditions, the size of the experiment, and the experimental material; and (3) an outline of the method of analysis and presentation of the results.

The statement of objectives should be concise and descriptive and may be in the form of questions to be answered, hypotheses to be tested, or effects to be estimated. The most common faults are vagueness and excessive ambition.

In the description of the experiment it is important to define clearly the environmental conditions whose effects are to be estimated and compared and to understand the role that each environmental condition will play in reaching the objectives. It should be determined whether the primary objective is merely a "screening of components" among the different environmental conditions or whether, in addition, it is desired to determine the fundamental aspects of the effects. Although it may be sufficient to conduct a screening test, it is often more practical to supply fundamental knowledge.

Finally, the draft should describe in some detail the proposed methods for drawing conclusions, for estimation, and for presentation of the results. Although these items constitute a valuable portion of the draft, they are the items

that are most frequently omitted. The draft may even include a sketch of the analysis to be used, an indication of the form in which the results will be shown, and some account of the precision expected in the effects to be estimated. Even if it is realized that an experiment must fall short of the precision desired, it is a good practice to try estimating the degree of precision that will be attained and to present this information as part of the proposal for the experiment. This process verifies which environmental conditions are relevant to each of the stated objectives of the experiment. Attention is drawn to deficiencies in the set of environmental conditions and to those conditions that supply little or no information essential to the purpose of the experiment.

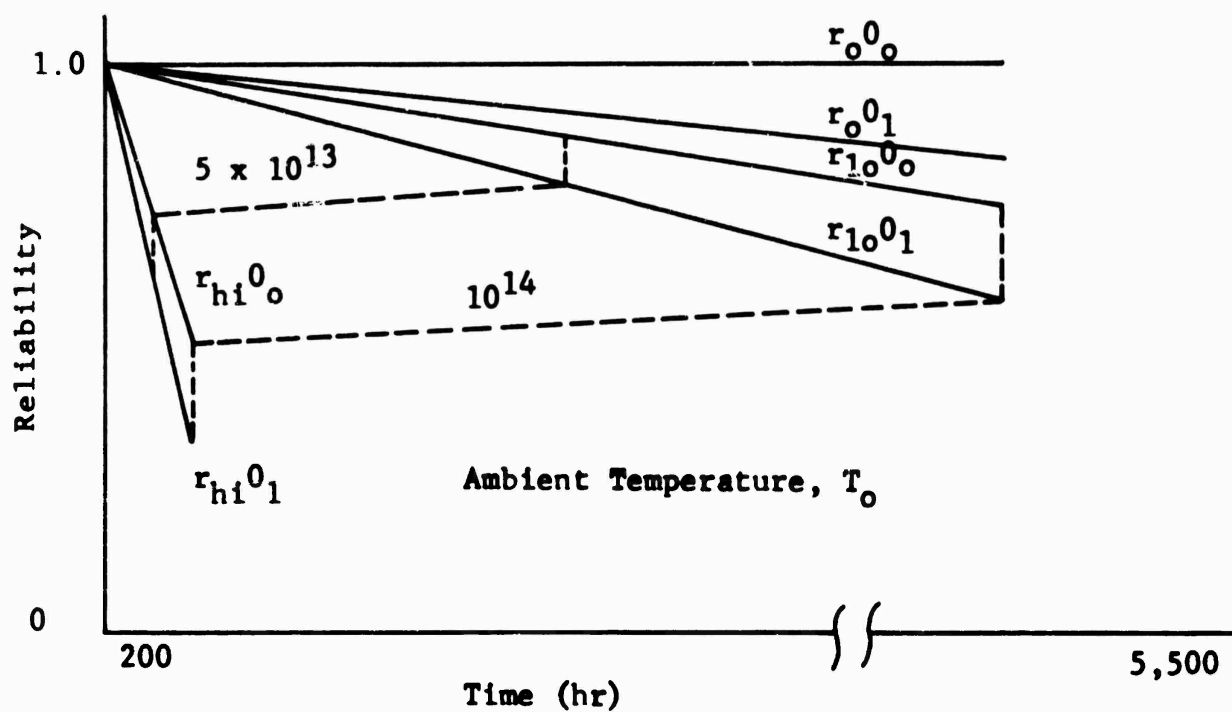
The brief comments and advice offered above have been directed towards some basic principles that are often overlooked. Further discussion on the subject, with respect to radiation experiments, statement of problem, manpower requirements, literature review, cost estimates, etc., are given in Reference 24.

In regard to the planning of experiments, the importance of understanding the role that each combination of environmental conditions will play in reaching the objectives of the experiment was pointed out. To reach this understanding requires a preanalysis to verify which environmental conditions are relevant to the stated objectives. There is much to be learned from an examination of the analysis methods and procedures used to evaluate the environmental conditions of the experiment. Therefore, figures and tables are given in the following pages to

demonstrate the methods and the problems one can and will encounter in the development of a realistic test plan.

The first step in the preanalysis is to examine every possible outcome of the experiment for the various environmental conditions and to draw conclusions for each possible outcome. Although all possible outcomes and possible conclusions are not presented herein, specific examples pertinent to the discussion are included. In the first cursory analysis an estimate of the rate effects and/or acceleration factors can be made by comparing the results of Block 11 with Block 9 in Figure 37. Further examination of $r_{10}T_0O_1$ for 5000 hr and $r_{h1}T_0O_1$ for 200 hr leads one to the conclusion that any observed difference (zero included) in the results could be the effect of rate, operation, or some combination of both. This conclusion, along with its various ramifications, is illustrated with the aid of Figure 38, a generic plot of some possible outcomes for various combinations of environments.

These data were constructed by assigning values to the environmental conditions of Blocks 1, 3, 5, and 7 of Figure 37; the other curves were then constructed by making an independent assumption concerning rate and/or operating effects. The possible outcomes considered from this set of data are shown in the following series of 2 x 3 arrays in Figure 39. The results listed below and tabulated in Figure 39a are for the environmental conditions from Blocks 1, 3, 5, and 7 of Figure 37. The values used in the discussion are deviations, as measured from the storage environment r_0O_0 , and are assumed to be zero.



- - - lines of equal nuclear exposure

$r_o^0_o$ is no radiation - non-operating (storage conditions)

$r_o^0_1$ is no radiation - operating conditions (Mfg's life-tests)

$r_{lo}^0_1$ is low-rate radiation - operating conditions, etc.

Figure 38 Generic Plot of Some Possible Outcomes for Various Combinations of Environments

	0_0	0_1	
r_o	0	-2	-2*
r_{lo}	-3	X=?	
r_{hi}	-6		

a.

	0_0	0_1	
r_o	0	-2	-2*
r_{lo}	-3	-5	-2
r_{hi}	-6	-8	-2

b.

	0_0	0_1	
r_o	0	-2	-2
r_{lo}	-3	-5	-2
r_{hi}	-6	-6	0

c.

	0_0	0_1	
r_o	0	-2	-2
r_{lo}	-6	-8	-2
r_{hi}	-6	-8	-2

d.

	0_0	0_1	
r_o	0	-2	-2
r_{lo}	-6	-8	-2
r_{hi}	-6	-6	0

e.

	0_0	0_1	
r_o	1	7	
r_{lo}	3	9	
r_{hi}	5	11	

Block No. Key

* The values in the margins are estimates of the operating effects at the various levels of radiation rates.

Figure 39 Sample Data to Demonstrate Preatalysis Procedures

From Figure 38, for ambient temperature T_o , the following values are considered in Figure 39a:

$$r_o^0 - r_o^0 = r_o^0 = -2, \text{ the operating effect for 5000 hr at no radiation, } r_o.$$

$$r_o^0 - r_o^0 = r_o^0 = 0 \text{ at 200 hr}$$

$$\text{for Block 3, } r_{lo}^0 = -3 \text{ and}$$

$$\text{for Block 5, } r_{hi}^0 = -6.$$

The results at the radiation conditions are compared at points of equal total exposure so that the difference between Blocks 3 and 5 is considered a rate effect. If no rate effect had been considered, the results at r_{lo}^0 would equal r_{hi}^0 , as in Figure 39d.

The construction of the tables in Figure 39 is based on certain assumptions concerning the independence of rate and operating effects. The radiation data are compared at points of equal total nuclear exposure for Blocks 3, 5, 9, and 11. To obtain equal exposures for the two different rates r_{lo} and r_{hi} , the point in time has to be chosen inversely proportional to the rates. In the case considered, Blocks 3 and 9 are for 5000 hr, and Blocks 5 and 11 are for 200 hr; for nonradiation, both the 200- and 5000-hr results can be considered.

Figure 39a contains the basic values selected for the discussion. The combination of environments at X, r_{lo}^0 (low-rate, operating, long-time) is that combination anticipated in application and is the most difficult to predict because of the long test times involved. In estimating the values for

Blocks 9 and 11 in Figure 39b, the assumption of independence of rate and operating effects is made. The operating effect of -2 was added to r_{10}^0 to give $r_{10}^1 = -3 + (-2) = -5$ (Fig. 39b). The rate effect of -3 obtained from $r_{hi}^1 - r_{10}^0 = -6 - (-3) = -3$ was added to r_{10}^1 to give $r_{hi}^1 = -5 + (-3) = -8$ (Fig. 39b); r_{hi}^0 could have been estimated by adding the operating effect, -2, to r_{hi}^0 to give $r_{hi}^1 = -6 + (-2) = -8$.

The assumption of independence of operating implies that the effect of operating for 200 hr is the same as that for 5000 hr; existing life-test data show that this is not the case. If the operating effect at 200 hr is negligible, as exhibited by the graphs in Figure 38, the results at r_{hi}^1 can be considered the same as in the nonoperating case, r_{hi}^0 , in Figure 39c; however, existing data show that in some cases this is not so. Figures 39d and 39e are constructed similar to Figures 39b and 39c, respectively, except that no rate effects are assumed; that is, the results at $r_{10}^0 =$ results at r_{hi}^0 , which implies that radiation damage is a function of total exposure only.

The above results lead to very puzzling circumstances that are not readily seen unless a detailed analysis is performed. The difficulty is in the interpretation, because the variable "time" interacts with rate and operating levels in such a way that the main effects cannot be estimated independently of each other; when time is increased, both total exposure and operating time are increased.

Some representative data from the manufacturer's life-tests and radiation-effects experiments are presented in Figure 40.

Although the magnitudes are not actual values, they do represent the sequence of events as they pertain to existing life-test and some radiation-effects experimental results.

The manufacturer's life-test data are identified as $r_o O_o$ - storage test at nonoperating conditions, $r_o O_{1/2}$ - life-test at, say, 50% rated capacity and $r_o O_1$ - life-test at 100% rated capacity. The life-tests are nominally 1000 hr in length. In this discussion, they are represented as 10,000 hr in order to be in the area of NAP application. As can be seen from the life-test results in Figure 40, the life-length of a component decreases with time as the operating level increases. This knowledge is used by designers to derate the component operating levels to increase the reliability of systems under consideration. The limited radiation-effects data that have been obtained for operating (active) and nonoperating (passive) conditions for different input levels indicate that some devices have a longer life-length under operating conditions as compared to nonoperating conditions. These results are exhibited by $r_{hi} O_o$ (high nuclear rate, nonoperating), $r_{hi} O_{1/2}$ (high nuclear rate, operating at 50% rated capacity), and $r_{hi} O_1$ (high nuclear rate, operating at 100% rated capacity). As indicated by the graph, there is a reversal of order in the effects of operating conditions from a nonradiation to a radiation environment. This raises an all important question: What will happen to the order of effects as the tests are run at lower rates and increasing times? In the existing data, only the extremes are present, r_{hi} and r_o ; no known data exist between these areas to indicate an answer. This

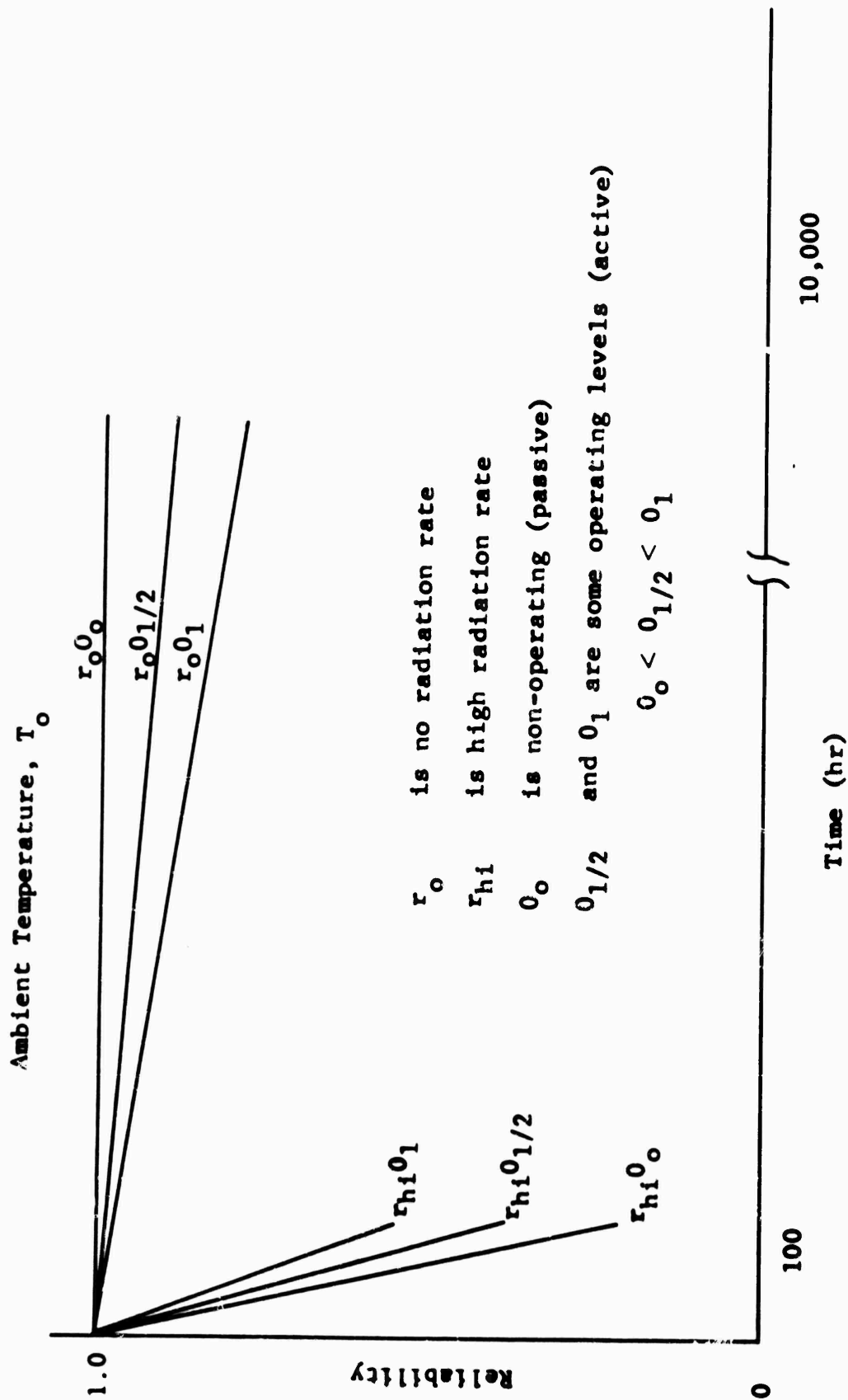


Figure 40 Some Representative Data from Manufacturer's Life-Tests and Radiation Effects Experiments

reversal in the order of operating effects indicates the presence of an interaction between rate, operation, and time. When interactions are present, main effects cease to have as much meaning, so that when an operating effect at a specified nuclear exposure is quoted, the levels of rate and time must be taken into account. An interesting consequence to this reversal problem is that there is some combination of rate, operating level, and time in which no operating effects are observed; otherwise the reversal is a sudden change.

Figure 41 is presented to illustrate the time sequence of some existing data with respect to the observed reversal in operating effects and to examine some possible outcomes as the radiation rate is decreased and time increased.

The zero values in Figure 41a are deviations as measured from $r_0 O_0$ for zero hours from start of test; in practice, these values will not all be zero because of the variation expected among similar devices. The values in Figure 41b show radiation and operating effects at high rates for 100 hr. These values show that the effects at high-rate nonoperating conditions are greater than the effects at high-rate operating conditions as is observed in some of the existing data. The value in the margin $[-6 - (-8) = +2]$ is interpreted as an increase of two units in the response characteristic due to use of an operating (active) device as compared with a nonoperating (passive) device. The 0 values indicate negligible effects at the low rate, r_{10} , conditions for 100 hr. The manufacturer's life-test data also show no effects at 100 hr. The values in Figure 41c show an

Pre-test			100 hr			10,000 hr		
	O_0	O_1		O_0	O_1		O_0	O_1
r_o	0	0		0	0		0	-2 -2
r_{lo}	0	0		0-	0-		X=?	
r_{hi}	0	0		-8	-6 +2			
a.			b.			c.		

	O_0	O_1	
r_o	0	-2 -2	
$r_{lo}(10,000 \text{ hr})$	-8	X=? ?	
$r_{hi}(100 \text{ hr})$	-8	-6 +2	
d.			

Figure 41 Time Sequence of Sample Data to Demonstrate Operating Effects in a Radiation Environment

operating effect of a -2 for 10,000 hr of operation as compared with nonoperating storage conditions. The problem, of course, is concerned with predicting the component response at X on the basis of the results at 100 hr of radiation and the 10,000-hr manufacturer's life-tests. Although no known data exist in this low-rate, long-time operating environment, it is still a good practice to examine the possible outcomes to evaluate whether or not prediction is feasible with existing information, and if not, to draw attention to deficiencies in existing information and to aid in directing research towards the problem areas.

The values in Figure 4ld are the combined values of Figures 4lb and 4lc with the addition of -8 at r_{10}^0 - which is based on the assumption that only the total-exposure effect is present at nonoperating conditions and which, in turn, implies no rate effects at nonoperating conditions. Of course, if there is a nonoperating effect, either as a main effect or as some interaction with rate and time, this value will be larger or smaller depending upon the nature of the effect; but for purposes of starting the discussion, this value, -8, will suffice. Then for the no-rate effects one has:

- Case 1 If the -2 operating effect at r_0 carries over to r_{10} , then $X = -8 + (-2) = -10$
- Case 2 If the +2 operating effect at r_{h1} carries over to r_{10} , then $X = -8 + 2 = -6$
- Case 3 If the effect is between -2 and +2, say 0, then $X = -8$.

These outcomes are all possible, and the interpretation and prediction is difficult. To demonstrate this, consider Table XIII,

where different assumptions have been made to obtain different values at r_{10}^0 , where $r_{10}^0 = -8$ is the value for Cases 1, 2, and 3 above.

Table XIII

SAMPLE DATA TO DEMONSTRATE A PROBLEM IN PREDICTING LONG-TERM, LOW-RADIATION-RATE RELIABILITY

<u>Assumed Values</u>	<u>Operating Effect</u>	<u>Value at X</u>
$r_{10}^0 = -6$	-2	-8
	+2	-4
	0	-6
$r_{10}^0 = -8$	-2	-10
	+2	-6
	0	-8
$r_{10}^0 = -10$	-2	-12
	+2	-8
	0	-10

Table XIII demonstrates the different outcomes that could be obtained in predicting results based on existing data and the various unverified assumptions. If the r_{10}^0 condition was not tested and only the -8 result in Table XIII was observed at X, for r_{10}^0 for 10,000 hr, there would be a one-third chance of guessing the correct conclusion. Even if the r_{10}^0 condition was tested, it is doubtful whether the result would add much to predicting X, because of the rate, operation, and time interaction. Note that assumptions have been made so that certain specific outcomes occur. When the value at X is observed from an experiment, the assumptions become possible conclusions. Thus, when three different assumptions can lead to the same result, there are three possible conclusions.

The lack of information for low-rate-radiation, long-time-test conditions precludes any definite statements as to the applicability of predicting results based on existing data. The presumption that an accelerated rate represents a more severe condition than will the lower rate of a 1-year SNAP exposure (so that it will provide an additional safety margin in reliability estimates) may not be so. If there are no rate effects (so only total exposure is the radiation damaging factor), the effects of time at temperature and/or time at operating level may result in additional damage and a decrease in reliability. This is shown in Figure 39e and, for certain values of X, in Table 13. The operating and temperature effects not observed in existing data may be the result of the short time (100 hr) high-rate radiation tests in which the radiation effect overshadows the temperature and operating effects. While at lower rates and longer times (10,000 hr), operating and temperature effects may play a role in reliability prediction and estimation that is as important as (or more important than) radiation effects. At this time, all the problems associated with low-rate, long-time tests are not known. Radiation effects observed in existing data may not be as critical, or may not even exist, at lower rates. For instance, some items now considered a poor risk may improve, whereas other items could get worse when operated for long-time missions. As in the initial stages of radiation experiments, when little was known about this new environment, the second law of experimentation was very much in evidence, i.e., "Anything that can happen will."

2. Data Presentation

Data presentation is actually the end-product of testing and analysis. Although no one method of presentation can do justice to the various types of information that can be anticipated, a proposed graphical approach patterned after the electronic manufacturers' presentations is a suggested method to adapt to the radiation-reliability data. The proven method of response-curve presentation as a function of applicable parameters, supported by a reliability plot (Weibull, log-normal, etc.) should be a practical layout. The response curve can be plotted as a "typical" curve of the sample of items tested. The "typical" curve may represent the median, mode, or arithmetic average, depending upon which fits the situation.

In addition to the "typical" curve, the maximum- and minimum-value curves should be included to show the range of possible values that have been observed in the data. The data should include the number of samples that were tested to make up the curves being presented. Individual data points may have to be plotted in some cases in which there are discontinuities, or in which the data variability is such that a smooth curve would not adequately represent the data. The primary objective should be to keep down the bulk and yet retain the content of the original data.

When the sample size permits, reliability plots (Weibull, log-normal, etc.) should be included to show the distribution of the data between the minimum and maximum curves for a range of failure criteria. When catastrophic failures occur, they

can be presented as reliability plots only. If catastrophic failures occur in combination with degradation failures (if identifiable) they can be noted on the response curves and plotted in combination with the degradation failures.

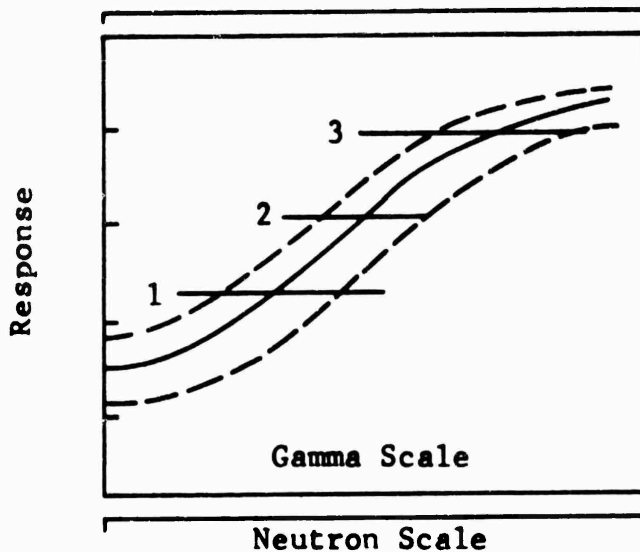
Figure 42 is an example of a proposed data presentation layout. The presentation is concerned with presenting compatible response curves and reliability plots for a range of practical failure criteria. It has been shown that even though a component is changing as a function of time, it is possible to design around predictable response-curve degradation. The designer thus requires a response curve to know how the device responds before and after it has been classed as a failure and to know the expected variation. The reliability curve will supply estimates and show the distribution of the data at the indicated failure criteria. All pertinent test information should be included on the data sheets.

Radiation-reliability plots with failure age in nuclear scales may be required if the test times between radiation tests and life-tests do not agree, as in the case of accelerated radiation tests, or if the radiation data are obtained at different rates. In these cases, a series of graphs may be required to represent the data.

Graphical presentation has much to offer over tables, in that the mind cannot grasp the significance of a list of numbers as readily as a visual picture represented by a graph. A standard format specified by the contracting agencies would enable one to make visual comparisons between various devices

Response Curve

Time Scale



Component _____

Mfg. _____

Inputs _____

Temperature _____

No. Tested _____

Failure Criteria

1. -

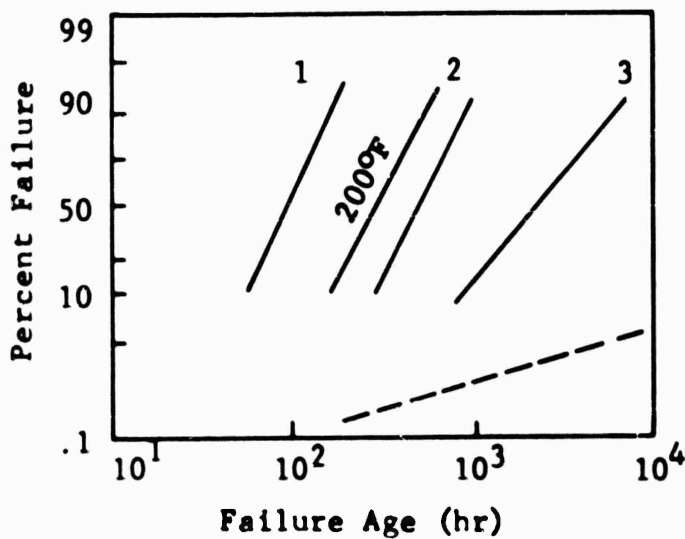
2. -

3. -

_____ Typical

_____ Min-Max

Radiation Reliability



Radiation Exposure

Neutron rate/hr _____

Gamma rate/hr _____

Remarks:

100°F test temperature
except where noted.

* - - - - 10,000-hr life-test

Inputs _____

Failure Criteria _____

Figure 42 Data Presentation Example

for selecting promising candidates for specific applications. In time the buildup of data will show whether the assumptions that have been made are holding up in practice.

Any presentation of data, no matter how enlightening, is wasted unless it has a broad dissemination among prospective users. Having a good product is fine, "but advertising helps." The manufacturer's method of dissemination is a proven procedure that has much to offer. Loose-leaf data sheets such as the example can be inserted into their respective binders and can be handled in the same way. Obsolete data can be discarded as up-to-date information becomes available.

3. Military Specifications

The need for an increase in the component life-test times and reliability will steadily increase with the anticipated increase in space mission times. The 20,000-hr (and more) missions will put a severe reliability requirement upon components. In this respect, military specifications have not kept pace with the anticipated applications. As one manufacturer put it, "We would go out of business if our product was geared to meet only the current 'mil specs.'" Therefore, a study should be made to up-date the present military specifications to keep pace with the anticipated requirements.

Some suggestions that can be made both from the results of this study and from the writings of others are:

- . Increase the test time
- . Record data at more intervals

- . Investigate time- and/or failure-truncated tests
- . Use a range of failure criteria on an individual basis and on a lot basis
- . Summarize data and report as graphical plots similar to design data presented by manufacturer (loose leaf)
- . Identify catastrophic and degradation failures

With an increase in test time, it follows that the number of recording times will have to be increased. An investigation into the possibilities of setting up time- and/or failure-truncated life-tests should be made. The test procedure would be to conduct the test to a specified time and a specified number of failures and to stop the test at the specified time or the specified number, whichever comes first.

A range of failure criteria would not change the test procedures, but would require additional analysis. An analysis of the data on an individual basis would identify those components classed as catastrophic and degradation failures. These data would be more easily compared with radiation life-test results.

Graphical presentation of the life-test data (failures) patterned after the example in Figure 42 would serve the same purpose as a radiation-reliability presentation. The actual failure data could be plotted, along with the estimates based on any assumptions that were made; in this way, it would be possible to have a visual verification of the assumptions made.

Although the graphical presentations do not give the statistical treatment desired, they can be used to demonstrate

whether the components, circuits, and systems are meeting specifications.

Further study of sampling plans should be made for defining quality. Such a study should be based on percentiles (reliability) instead of mean life, especially for radiation-reliability specifications.

SECTION VI

CONCLUSIONS

The purpose of this study was to determine the relationship between radiation effects and reliability of space-systems components and to investigate the applicability of various methods of predicting component reliability in complex environments. The study necessarily covered various subjects, and numerous conclusions are made on each subject. The most prominent conclusions or results for each subject are outlined below.

1. Applicability of Standard Statistical Technology to Radiation Reliability

An understanding of basic statistical methods by the experimenter is necessary in the performance of reliability studies.

Radiation is simply an additional environment, although a complex one, and can be handled by classical statistical methods.

Although classical methods can be used in radiation reliability, reliability technology - especially for long-term missions - needs further development and refinement. Such items as analysis by computer methods, mathematical models for radiation reliability, and data-handling procedures should be developed and qualified for predicting system reliability.

The Weibull distribution is a versatile failure model for approximating and expressing radiation reliability. The Weibull distribution fits the radiation data examined for this study better than the exponential distribution.

2. Sensitivity of Available Data

The data analyzed in the study are primarily from manufacturer's life-tests, in which radiation effects were not considered, and from short-term radiation experiments, in which reliability was not part of the test objective. Some conclusions that can be made with respect to each type of datum are:

a. Manufacturer's Life-Test Data

The shape parameter, β , of the Weibull distribution is predominantly less than unity.

An increase in temperature causes a decrease in the lifetime of the electronic components analyzed.

An increase in the operating level causes a decrease in the lifetime of the components.

The failures observed are primarily a degradation type in which there is a gradual rather than a catastrophic change.

b. Radiation Effects Data

The shape parameter, β , of the Weibull distribution is predominantly greater than unity.

The radiation lifetime generally increases with increasing temperature.

An increase in the operating level increases the radiation lifetime of some devices and decreases the radiation lifetime of others.

The failures observed are primarily a degradation type in which there is a gradual rather than a catastrophic change.

3. Long-Term-Reliability Test Techniques

Analysis of available data shows that the interactions of radiation rate, operating conditions, and temperature are too complex to extrapolate 10C-hr data to 10,000-hr predictions.

Accelerated-test methods should be possible with development of explicit test techniques and mathematical models.

The importance of eliminating a series of long-term tests in support of long-term missions warrants extensive effort in development of accelerated test techniques.

4. Methods of Data Presentation

Reliability data should be presented for a range of practical failure criteria.

Most reliability data are presented in a form based on the unverified exponential assumption; these data would prove more useful if the actual failure data were presented as probability plots.

Since reliability is becoming more important with longer mission time requirements, reliability data should be made available on the same basis as the standard manufacturer's design data.

5. Military Specifications for Long-Term Missions

Military specifications have not kept pace with anticipated long-term applications and should be improved as soon as possible to allow timely completion of future long-term missions.

Reliability test times should be increased.

Reliability data should be recorded at more frequent intervals.

Catastrophic and degradation failures should be identified as such.

A range of failure criteria should be presented on both an individual basis and a lot basis.

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13 ABSTRACT The applicability of standard mathematical techniques for analysis of reliability of components exposed to a radiation environment is presented. The sensitivity of failure-distribution functions, data-presentation techniques, statistical parameters, and types of measurements to practical analysis methods is demonstrated. With the insight gained from the analysis methods, a research and development program was designed to establish qualified test procedures for long-term nuclear-development techniques. In addition, the lack of practical standard test techniques is substantiated and standard data-reporting methods are recommended.		

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